SOC 3811/5811:
BASIC SOCIAL STATISTICS

Analysis of Variance

Associations Between Variables

Between now and the 3rd exam we will focus on measuring the association between two variables, X & Y

1. When X is discrete and Y is continuous, we will use "analysis of variance" techniques (Today)
2. When X and Y are both discrete, we will use cross-tabular and χ^2 analyses (Thursday)
3. When X and Y are both continuous, we will use correlation & regression analyses (Next Week)

ANALYSIS OF VARIANCE

Introduction to ANOVA

ANalysis **O**f **VA**riance (**ANOVA**) techniques compare the mean of continuous variable Y across *J* populations

Such questions amount to comparing the mean of some continuous variable Y across the *J* categories of some discrete variable X

ANOVA amounts to a test of the hypothesis that all of the *J* population means are equal:

H_0: $\mu_1=\mu_2=\mu_3=\ldots=\mu_J$

H_1: Not all of the means are equal

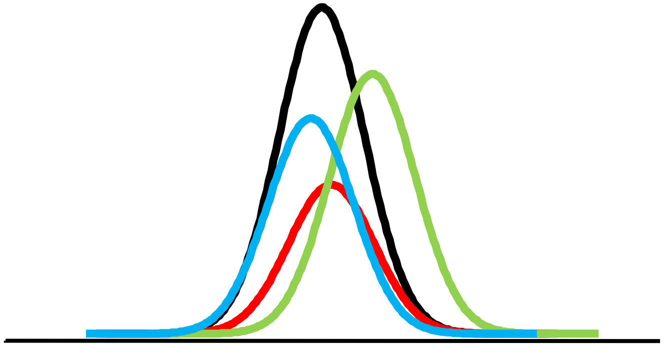
Introduction to ANOVA

The test statistic that we use to perform the ANOVA hypothesis test is called the *F* statistic

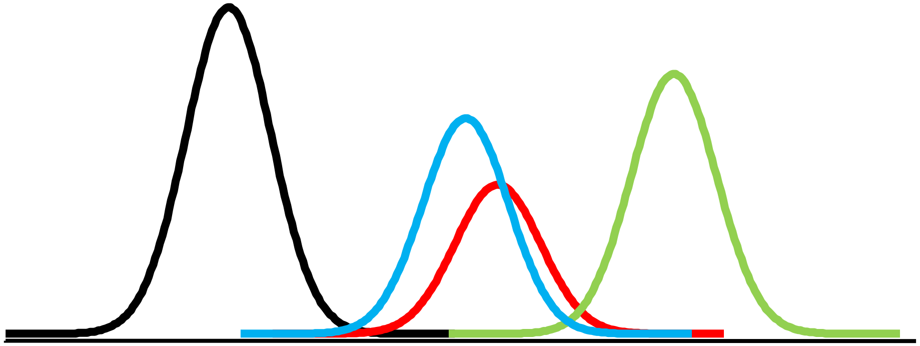
Conceptually, the more the *J* sample means differ, the larger the *F* statistic

If *F* is larger than we would expect by chance … if the observed value of the *F* test statistic exceeds a critical value determined in advance … then we reject H_0

Introduction to ANOVA

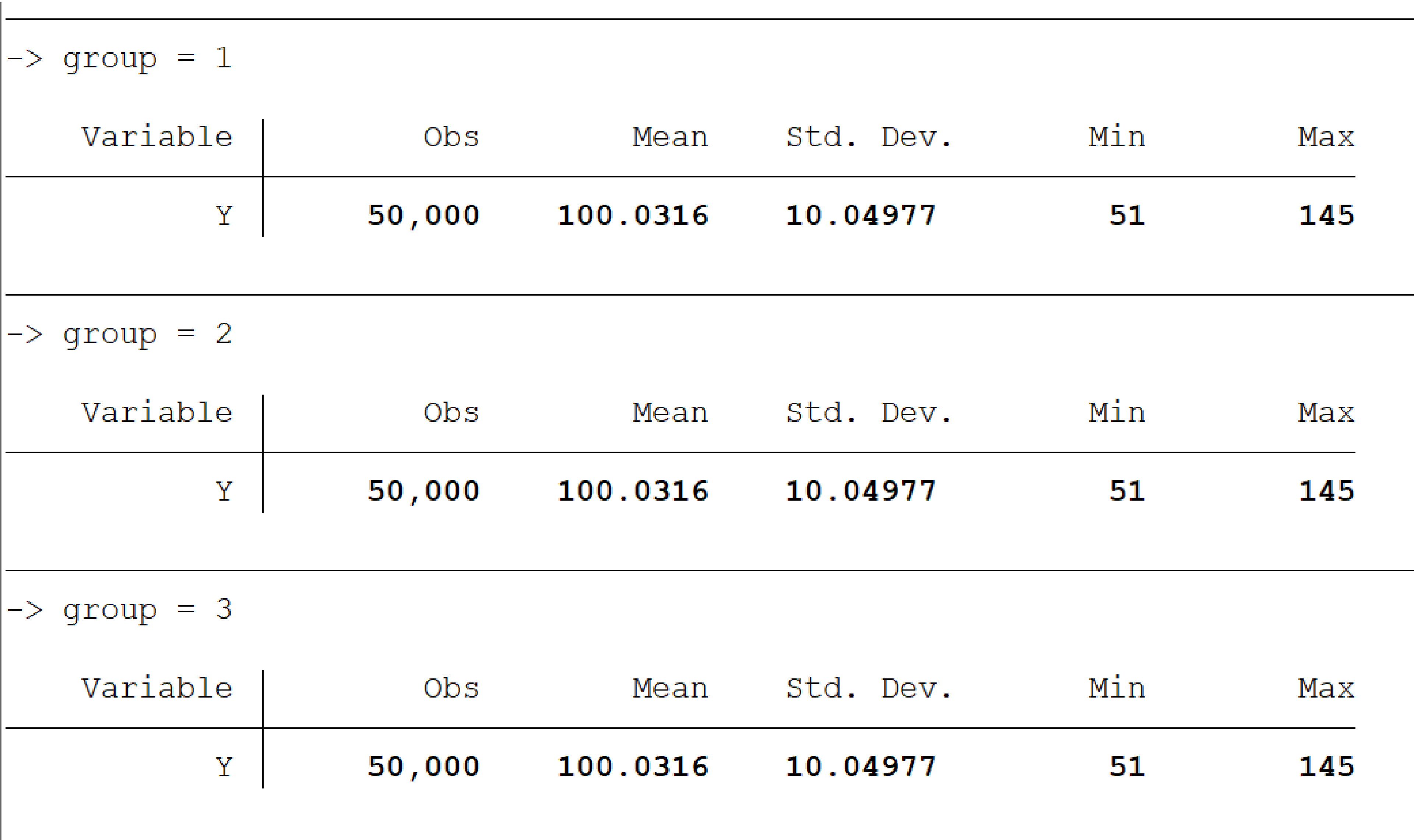


Small *F* Value



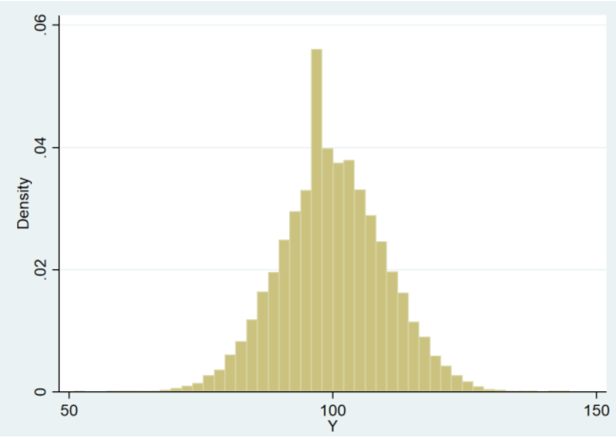
Larger *F* Value

The F Statistic

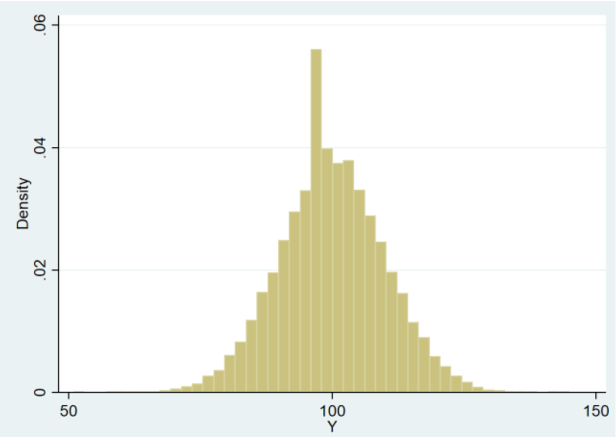


```
-> group = 1

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
           Y |     50,000    100.0316    10.04977         51        145

-> group = 2

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
           Y |     50,000    100.0316    10.04977         51        145

-> group = 3

    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
           Y |     50,000    100.0316    10.04977         51        145
```

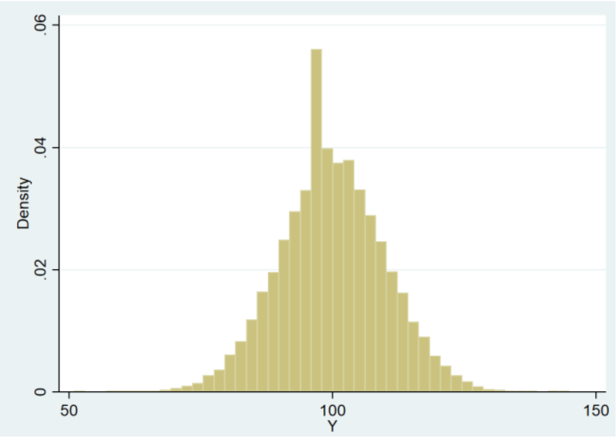
The F Statistic



Group 1

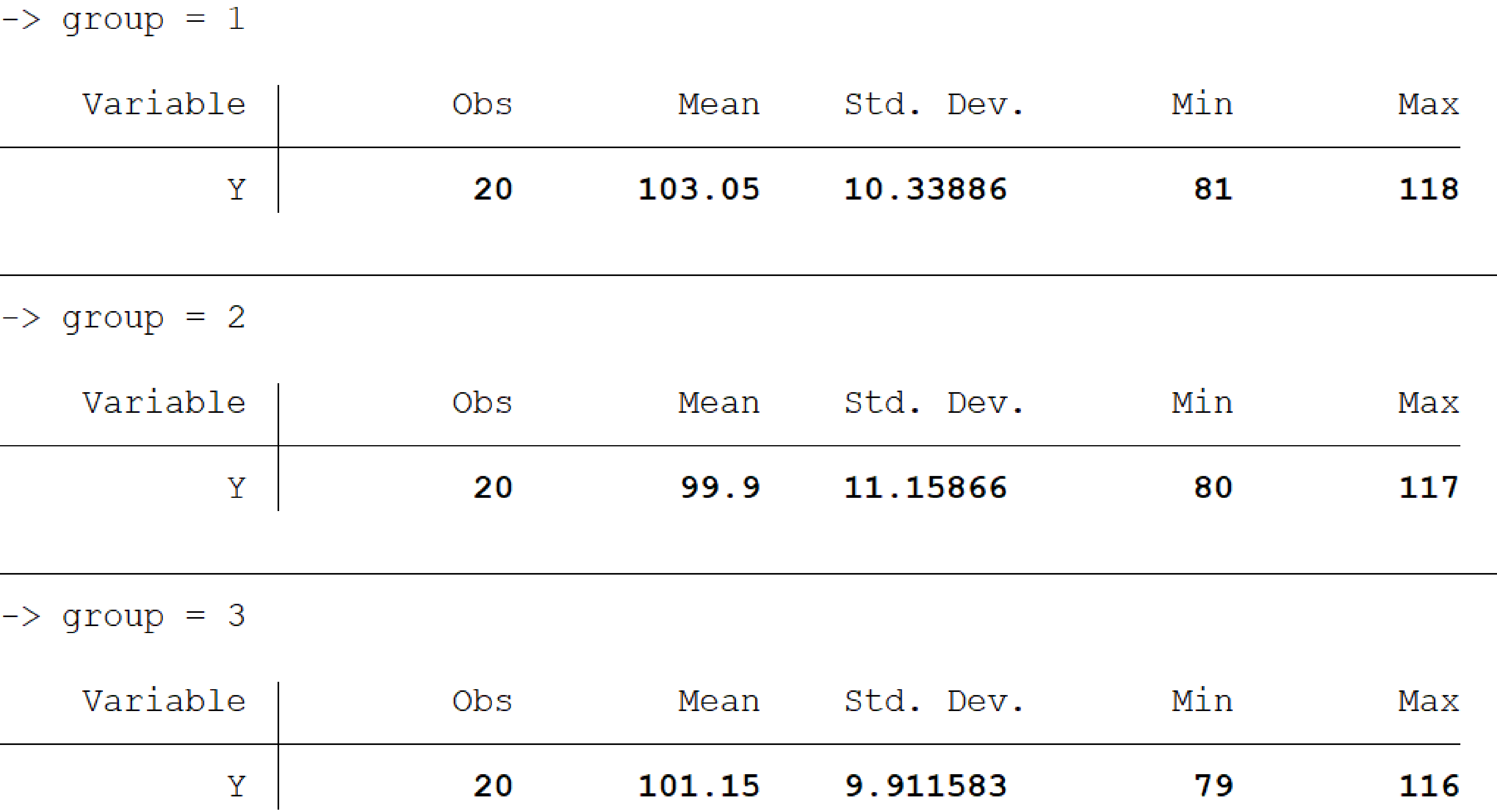


Group 2



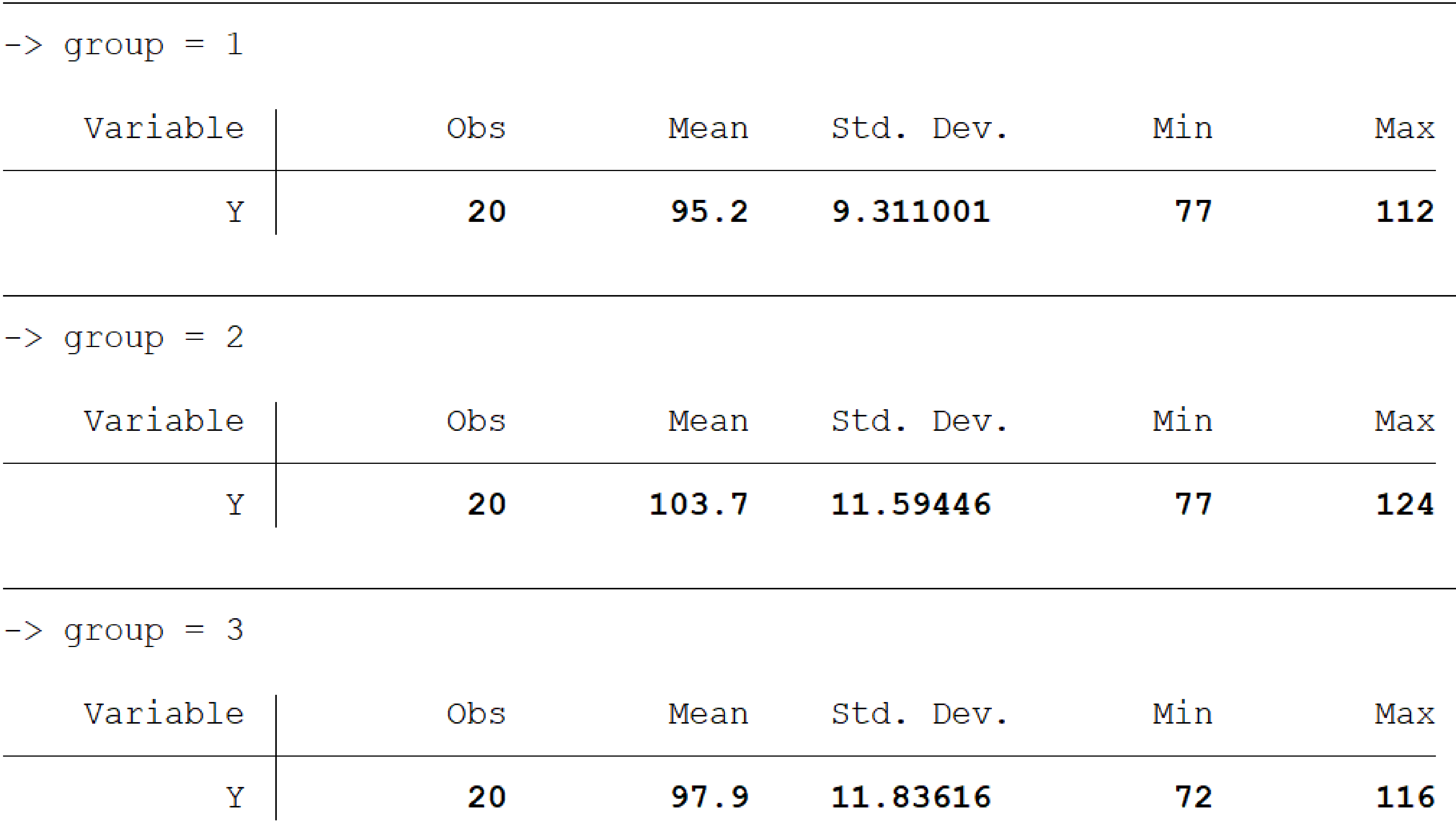
Group 3

The F Statistic



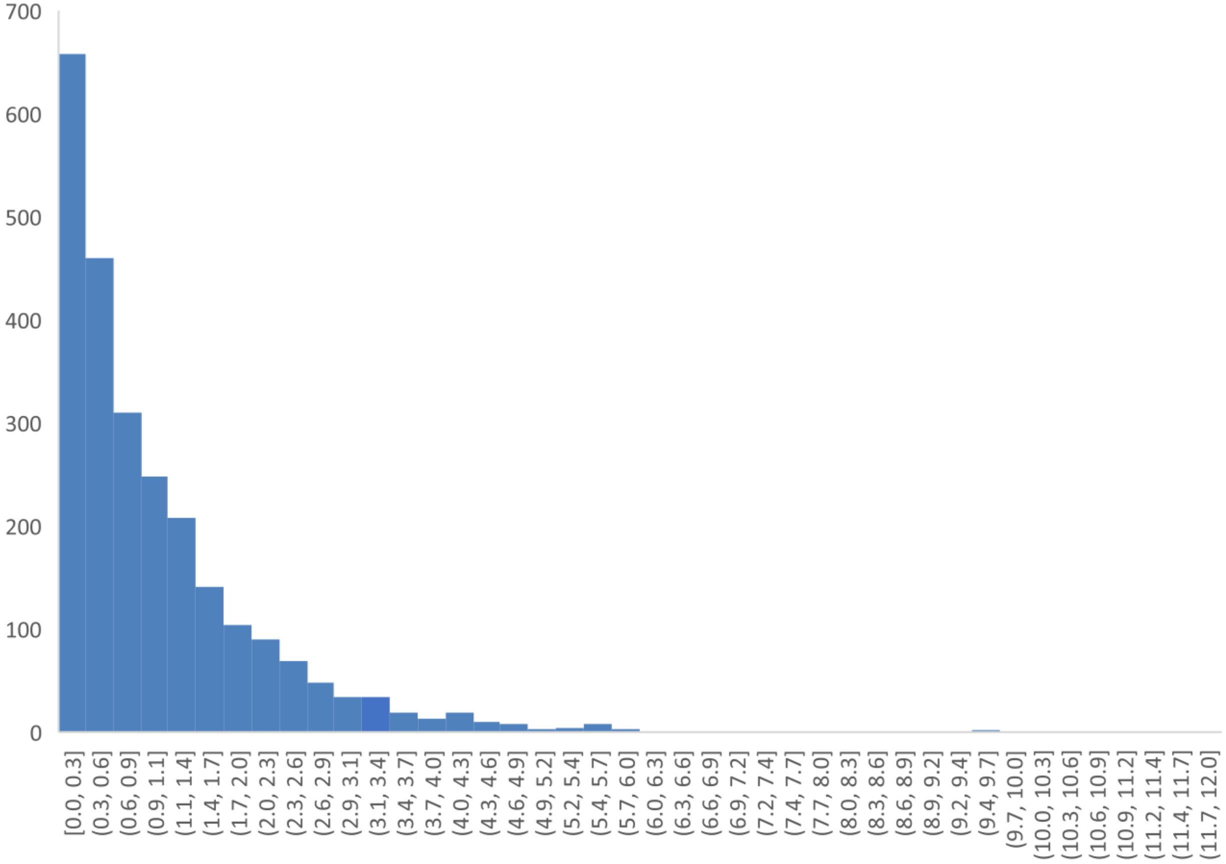
```
-> group = 1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           Y |        20      103.05    10.33886         81        118

-------------------------------------------------------------------------
-> group = 2

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           Y |        20        99.9    11.15866         80        117

-------------------------------------------------------------------------
-> group = 3

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           Y |        20      101.15    9.911583         79        116
```

The F Statistic

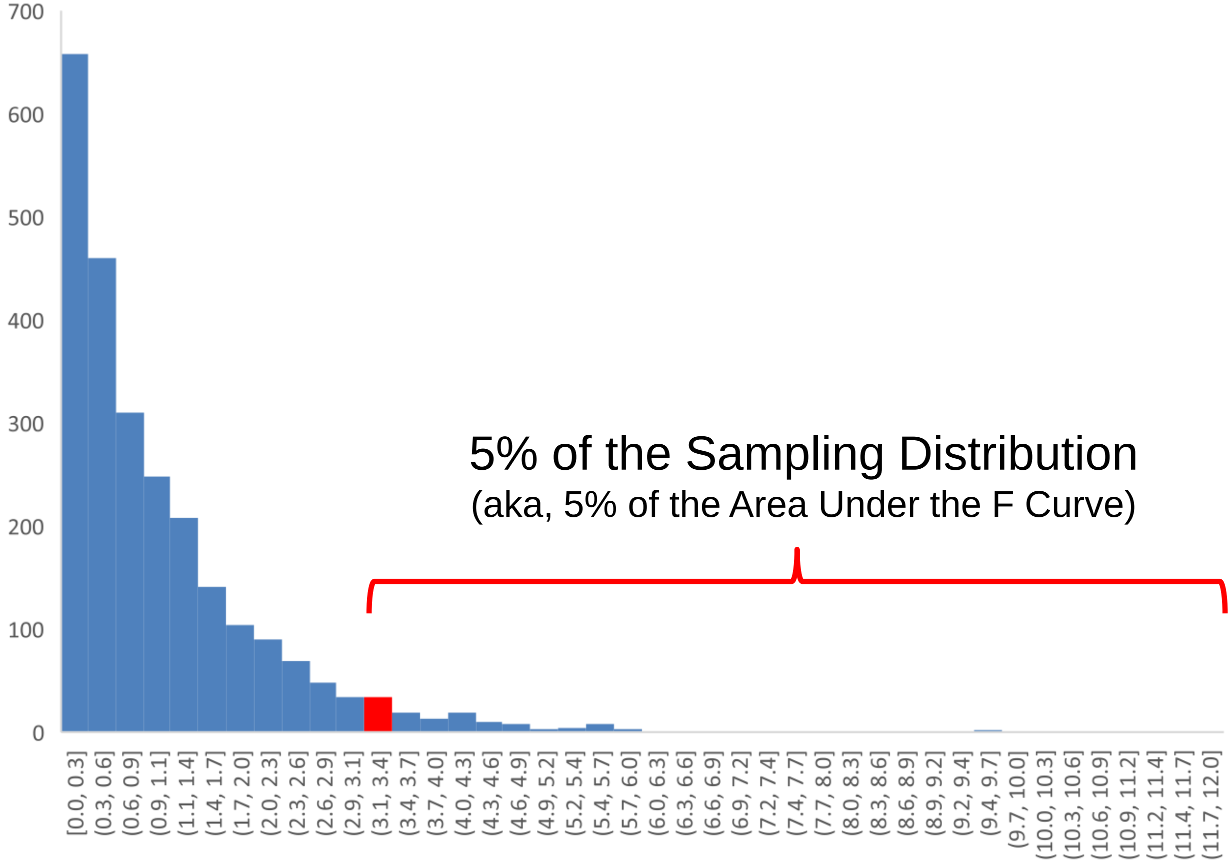


```
-> group = 1

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           Y |        20        95.2    9.311001         77        112

-> group = 2

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           Y |        20       103.7    11.59446         77        124

-> group = 3

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           Y |        20        97.9    11.83616         72        116
```

The F Statistic



The F Statistic



5% of the Sampling Distribution

(aka, 5% of the Area Under the F Curve)

ANOVA Example

Does employee productivity—how many minutes is takes them to perform a task—vary by how they are trained to do the task?

To find out, a company randomly assigned 60 new employees to get either Training A, Training B, or no training at all and then recorded the number of minutes it took the employees to complete the task.

Using $\alpha=0.05$, assess whether the time it takes to complete the task varies by training type in the population

ANOVA Example

Below are the mean, standard deviation, and 95% confidence intervals for the three groups:

Training	**Mean**	**s.d.**	**95% C.I.**	**n**
Training A	12.9	3.0	11.5 to 14.3	20
Training B	9.0	2.6	7.8 to 10.2	20
No Training	14.7	3.4	13.1 to 16.2	20

ANOVA Example

Hypothesis Testing in 6 Steps … Just Like Before

1. State the null (H_0) and alternative (H_1) hypotheses
2. Check that the sample data conform to basic assumptions; if they do not, then do not go any further
3. Choose an α probability level … that is, a probability associated with incorrectly rejecting the null hypothesis
4. Determine the "critical value" … that is, how large the test statistic must be in order to reject the null hypothesis at the given α level
5. Calculate the test statistic ... F
6. Compare the test statistic to the critical value

ANOVA Example

State the null (H_0) and alternative (H_1) hypotheses

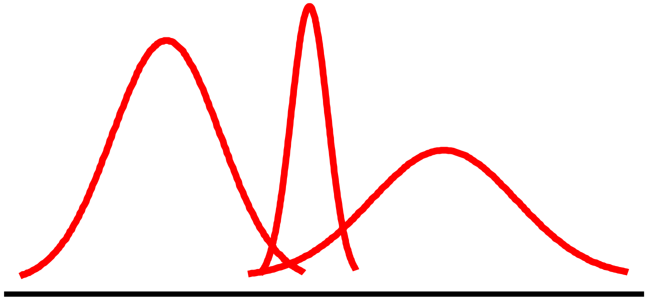
H_0: $\mu_{\text{Training A}} = \mu_{\text{Training B}} = \mu_{\text{No Training}}$

H_1: Not all of the population group means are equal

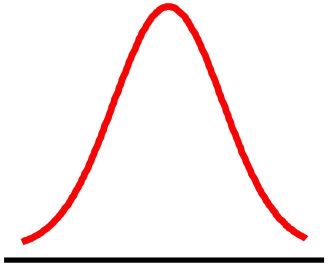
ANOVA Example

Check that the sample data conform to basic assumptions; if they do not, then do not go any further

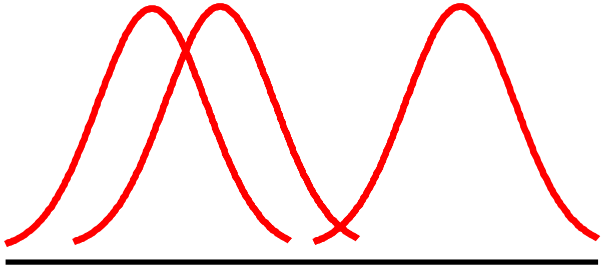
1. The *j* samples are independent random samples
2. Within each population group, Y is normally distributed
3. The standard deviation of Y is equal across the *j* population groups (the "homoskedasticity" assumption)



Assumptions Are Not Met



H_0: $\mu_1=\mu_2=\mu_3$



H_1: Not all Means Are Equal

ANOVA Example

Choose an α probability level … that is, a probability associated with incorrectly rejecting the null hypothesis

α=0.05

Determine the “critical value” … that is, how large the test statistic must be in order to reject the null hypothesis at the given α level

The critical value of F depends on α, df_{NUM} (which equals J-1), and df_{DENOM} (which equals N-J). In our example, N=60 and J=3, so df_{NUM}=2 and df_{DENOM}=57

Critical Values of F

(α=0.05)

DENOMINATOR Degrees of Freedom	NUMERATOR Degrees of Freedom																	
	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	100	200	∞
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	245.95	248.01	250.10	251.14	251.77	253.04	253.68	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45	19.46	19.47	19.48	19.49	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.62	8.59	8.58	8.55	8.54	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.75	5.72	5.70	5.66	5.65	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.50	4.46	4.44	4.41	4.39	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.81	3.77	3.75	3.71	3.69	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.38	3.34	3.32	3.27	3.25	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.08	3.04	3.02	2.97	2.95	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.86	2.83	2.80	2.76	2.73	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.70	2.66	2.64	2.59	2.56	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.57	2.53	2.51	2.46	2.43	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.47	2.43	2.40	2.35	2.32	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.38	2.34	2.31	2.26	2.23	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.31	2.27	2.24	2.19	2.16	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.25	2.20	2.18	2.12	2.10	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.19	2.15	2.12	2.07	2.04	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.15	2.10	2.08	2.02	1.99	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.11	2.06	2.04	1.98	1.95	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.07	2.03	2.00	1.94	1.91	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.04	1.99	1.97	1.91	1.88	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.18	2.10	2.01	1.96	1.94	1.88	1.84	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	1.98	1.94	1.91	1.85	1.82	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.13	2.05	1.96	1.91	1.88	1.82	1.79	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.94	1.89	1.86	1.80	1.77	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.09	2.01	1.92	1.87	1.84	1.78	1.75	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.90	1.85	1.82	1.76	1.73	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.06	1.97	1.88	1.84	1.81	1.74	1.71	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.87	1.82	1.79	1.73	1.69	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.03	1.94	1.85	1.81	1.77	1.71	1.67	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.84	1.79	1.76	1.70	1.66	1.62
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	2.00	1.92	1.83	1.78	1.75	1.68	1.65	1.61
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	1.99	1.91	1.82	1.77	1.74	1.67	1.63	1.59
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	1.98	1.90	1.81	1.76	1.72	1.66	1.62	1.58
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	1.97	1.89	1.80	1.75	1.71	1.65	1.61	1.57
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	1.96	1.88	1.79	1.74	1.70	1.63	1.60	1.56
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.74	1.69	1.66	1.59	1.55	1.51
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.87	1.78	1.69	1.63	1.60	1.52	1.48	1.44
75	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96	1.80	1.71	1.61	1.55	1.52	1.44	1.39	1.34
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.77	1.68	1.57	1.52	1.48	1.39	1.34	1.28
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.72	1.62	1.52	1.46	1.41	1.32	1.26	1.19
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.67	1.57	1.46	1.39	1.35	1.24	1.17	1.00

ANOVA Example

In our example, with α=0.05, the critical value of F is 3.18

We can thus state a decision rule:

H_0: $\mu_{\text{Training A}} = \mu_{\text{Training B}} = \mu_{\text{No Training}}$
H_1: Not all of the population group means are equal

Fail to reject H_0 if $F \leq 3.18$
Reject H_0 if $F > 3.18$

ANOVA Example

Calculate the test statistic ... *F*

$$s^2 = \frac{\sum (Y - \bar{Y})^2}{n-1}$$

$SS_{Between}$ = 334.6 with J-1=2 df

SS_{Within} = 520.35 with N-J = 57 df

$$F_{J-1,N-J} = \frac{SS_{BETWEEN}/J-1}{SS_{WITHIN}/N-J} = \frac{334.6/2}{520.35/57} = 18.33$$

ANOVA Example

Calculate the test statistic ... *F*

$\text{SS}_{\text{Between}}$ = 334.6 with J-1=2 df

$\text{SS}_{\text{Within}}$ = 520.35 with N-J = 57 df

$$F_{J-1,N-J} = \frac{\text{SS}_{\text{BETWEEN}}/\text{J}-1}{\text{SS}_{\text{WITHIN}}/\text{N}-\text{J}} = \frac{334.6/2}{520.35/57} = 18.33$$

ANOVA Example

Compare the test statistic to the critical value

1. If the test statistic is larger than the critical value, then reject H_0 (with probability of α of doing so even though H_0 should not be rejected)
2. If the test statistic is less than or equal to the critical value, then do no reject H_0 (with probability of β of doing so even though H_0 should be rejected)

We determined that our critical value of *F* is 3.18

We observed an *F* statistic of 18.33

H_0: $F \leq 3.18$ H_1: $F > \underline{3.18}$

Conclusion: Reject H_0

ANOVA Example

Below are the mean, standard deviation, and 95% confidence intervals for the three groups:

Training	Mean	s.d.	95% C.I.	n
Training A	12.9	3.0	11.5 to 14.3	20
Training B	9.0	2.6	7.8 to 10.2	20
No Training	14.7	3.4	13.1 to 16.2	20

F Table Practice

With α=0.05, what is the critical value of *F* when…

df_{NUM}=7, and df_{DENOM}=80 ?

df_{NUM}=20, and df_{DENOM}=2,400 ?

With α=0.01, what is the critical value of *F* when…

df_{NUM}=8, and df_{DENOM}=982 ?

df_{NUM}=14, and df_{DENOM}=30 ?

Critical Values of F
(α=0.05)

	NUMERATOR Degrees of Freedom																	
DENOMINATOR Degrees of Freedom	1	2	3	4	5	6	7	8	9	10	15	20	30	40	50	100	200	∞
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	245.95	248.01	250.10	251.14	251.77	253.04	253.68	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45	19.46	19.47	19.48	19.49	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.62	8.59	8.58	8.55	8.54	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.75	5.72	5.70	5.66	5.65	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.50	4.46	4.44	4.41	4.39	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.81	3.77	3.75	3.71	3.69	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.38	3.34	3.32	3.27	3.25	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.08	3.04	3.02	2.97	2.95	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.86	2.83	2.80	2.76	2.73	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.70	2.66	2.64	2.59	2.56	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.57	2.53	2.51	2.46	2.43	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.47	2.43	2.40	2.35	2.32	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.38	2.34	2.31	2.26	2.23	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.31	2.27	2.24	2.19	2.16	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.25	2.20	2.18	2.12	2.10	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.19	2.15	2.12	2.07	2.04	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.15	2.10	2.08	2.02	1.99	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.11	2.06	2.04	1.98	1.95	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.07	2.03	2.00	1.94	1.91	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.04	1.99	1.97	1.91	1.88	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.18	2.10	2.01	1.96	1.94	1.88	1.84	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	1.98	1.94	1.91	1.85	1.82	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.13	2.05	1.96	1.91	1.88	1.82	1.79	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.94	1.89	1.86	1.80	1.77	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.09	2.01	1.92	1.87	1.84	1.78	1.75	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.90	1.85	1.82	1.76	1.73	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.06	1.97	1.88	1.84	1.81	1.74	1.71	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.87	1.82	1.79	1.73	1.69	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.03	1.94	1.85	1.81	1.77	1.71	1.67	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.84	1.79	1.76	1.70	1.66	1.62
31	4.16	3.30	2.91	2.68	2.52	2.41	2.32	2.25	2.20	2.15	2.00	1.92	1.83	1.78	1.75	1.68	1.65	1.61
32	4.15	3.29	2.90	2.67	2.51	2.40	2.31	2.24	2.19	2.14	1.99	1.91	1.82	1.77	1.74	1.67	1.63	1.59
33	4.14	3.28	2.89	2.66	2.50	2.39	2.30	2.23	2.18	2.13	1.98	1.90	1.81	1.76	1.72	1.66	1.62	1.58
34	4.13	3.28	2.88	2.65	2.49	2.38	2.29	2.23	2.17	2.12	1.97	1.89	1.80	1.75	1.71	1.65	1.61	1.57
35	4.12	3.27	2.87	2.64	2.49	2.37	2.29	2.22	2.16	2.11	1.96	1.88	1.79	1.74	1.70	1.63	1.60	1.56
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.74	1.69	1.66	1.59	1.55	1.51
50	4.03	3.18	2.79	2.56	2.40	2.29	2.20	2.13	2.07	2.03	1.87	1.78	1.69	1.63	1.60	1.52	1.48	1.44
75	3.97	3.12	2.73	2.49	2.34	2.22	2.13	2.06	2.01	1.96	1.80	1.71	1.61	1.55	1.52	1.44	1.39	1.34
100	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.77	1.68	1.57	1.52	1.48	1.39	1.34	1.28
200	3.89	3.04	2.65	2.42	2.26	2.14	2.06	1.98	1.93	1.88	1.72	1.62	1.52	1.46	1.41	1.32	1.26	1.19
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.67	1.57	1.46	1.39	1.35	1.24	1.17	1.00

Critical Values of F
(α=0.01)

	NUMERATOR Degrees of Freedom																	
DENOMINATOR Degrees of Freedom	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**	**20**	**30**	**40**	**50**	**100**	**200**	**∞**
1	4052.18	4999.50	5403.35	5624.58	5763.65	5858.99	5928.36	5981.07	6022.47	6055.85	6157.28	6208.73	6260.65	6286.78	6302.52	6334.11	6349.97	6365.86
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.43	99.45	99.47	99.47	99.48	99.49	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	26.87	26.69	26.50	26.41	26.35	26.24	26.18	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02	13.84	13.75	13.69	13.58	13.52	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.72	9.55	9.38	9.29	9.24	9.13	9.08	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.56	7.40	7.23	7.14	7.09	6.99	6.93	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.31	6.16	5.99	5.91	5.86	5.75	5.70	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.52	5.36	5.20	5.12	5.07	4.96	4.91	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.96	4.81	4.65	4.57	4.52	4.41	4.36	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.56	4.41	4.25	4.17	4.12	4.01	3.96	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.25	4.10	3.94	3.86	3.81	3.71	3.66	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.01	3.86	3.70	3.62	3.57	3.47	3.41	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.82	3.66	3.51	3.43	3.38	3.27	3.22	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.66	3.51	3.35	3.27	3.22	3.11	3.06	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.52	3.37	3.21	3.13	3.08	2.98	2.92	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.41	3.26	3.10	3.02	2.97	2.86	2.81	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.31	3.16	3.00	2.92	2.87	2.76	2.71	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.23	3.08	2.92	2.84	2.78	2.68	2.62	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.15	3.00	2.84	2.76	2.71	2.60	2.55	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.09	2.94	2.78	2.69	2.64	2.54	2.48	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.03	2.88	2.72	2.64	2.58	2.48	2.42	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	2.98	2.83	2.67	2.58	2.53	2.42	2.36	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	2.93	2.78	2.62	2.54	2.48	2.37	2.32	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	2.89	2.74	2.58	2.49	2.44	2.33	2.27	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.85	2.70	2.54	2.45	2.40	2.29	2.23	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.81	2.66	2.50	2.42	2.36	2.25	2.19	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.78	2.63	2.47	2.38	2.33	2.22	2.16	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.75	2.60	2.44	2.35	2.30	2.19	2.13	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.73	2.57	2.41	2.33	2.27	2.16	2.10	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.70	2.55	2.39	2.30	2.25	2.13	2.07	2.01
31	7.53	5.36	4.48	3.99	3.67	3.45	3.28	3.15	3.04	2.96	2.68	2.52	2.36	2.27	2.22	2.11	2.04	1.98
32	7.50	5.34	4.46	3.97	3.65	3.43	3.26	3.13	3.02	2.93	2.65	2.50	2.34	2.25	2.20	2.08	2.02	1.96
33	7.47	5.31	4.44	3.95	3.63	3.41	3.24	3.11	3.00	2.91	2.63	2.48	2.32	2.23	2.18	2.06	2.00	1.93
34	7.44	5.29	4.42	3.93	3.61	3.39	3.22	3.09	2.98	2.89	2.61	2.46	2.30	2.21	2.16	2.04	1.98	1.91
35	7.42	5.27	4.40	3.91	3.59	3.37	3.20	3.07	2.96	2.88	2.60	2.44	2.28	2.19	2.14	2.02	1.96	1.89
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.52	2.37	2.20	2.11	2.06	1.94	1.87	1.80
50	7.17	5.06	4.20	3.72	3.41	3.19	3.02	2.89	2.78	2.70	2.42	2.27	2.10	2.01	1.95	1.82	1.76	1.68
75	6.99	4.90	4.05	3.58	3.27	3.05	2.89	2.76	2.65	2.57	2.29	2.13	1.96	1.87	1.81	1.67	1.60	1.52
100	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.22	2.07	1.89	1.80	1.74	1.60	1.52	1.43
200	6.76	4.71	3.88	3.41	3.11	2.89	2.73	2.60	2.50	2.41	2.13	1.97	1.79	1.69	1.63	1.48	1.39	1.28
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.04	1.88	1.70	1.59	1.52	1.36	1.25	1.00

Another ANOVA Example

Do the populations of people who live in cities, in the suburbs, and in rural areas own different numbers of cars? To find out, I randomly sampled 5 urban residents, 5 suburban residents, and 5 rural residents; I asked each person how many cars they own. Here are the responses I obtained:

y = # Cars

J=3

Urban residents:	0	1	1	0	0 ($\bar{y}_{\text{urban}}$= 0.400; s_{urban} = 0.548)
Suburban residents:	2	1	1	2	1 ($\bar{y}_{\text{suiburban}}$= 1.400 ;$s_{\text{suburban}}$ = 0.548)
Rural residents:	2	2	3	1	2 ($\bar{y}_{\text{rural}}$= 2.000; s_{rural} = 0.707)

For the full sample: $\bar{y}$ = 1.267, SS_{Between}=3.92 and SS_{Within}=4.40

Test the hypothesis that in the population the mean number of cars owned is the same for people who live in urban, suburban, and rural areas. Use a significance level of α = 0.05. Be sure to state the competing hypotheses, compute the appropriate test statistic, and state your conclusion about whether you reject or fail to reject H_0. *(Here, ignore the assumptions that (1) the variable must be distributed normally within each population and (2) that the amount of variability in each group must be the same. It's just a practice example. If this were real research, of course, we would not do so.)*

$H_0: \mu_R = \mu_S = \mu_U$ $\alpha = .05$

H_a : N γ

Critical Value $F_{J-1,N-J} = F_{2,12} = 3.89$

Reject H_0 if $F > \boxed{3.89}$

$$F = \boxed{\frac{\boxed{SS_{BET}}/J-1}{\boxed{SS_{WITHIN}}/N-J}} = \frac{3.92/2}{4.40/12} = \boxed{\underline{5.345}}$$

Reject H_0

$\times$ ALL ELSE

$$S_y^2 = \frac{\boxed{\sum(y-\bar{y})^2}}{n-1}$$