# SOC 3811/5811: BASIC SOCIAL STATISTICS

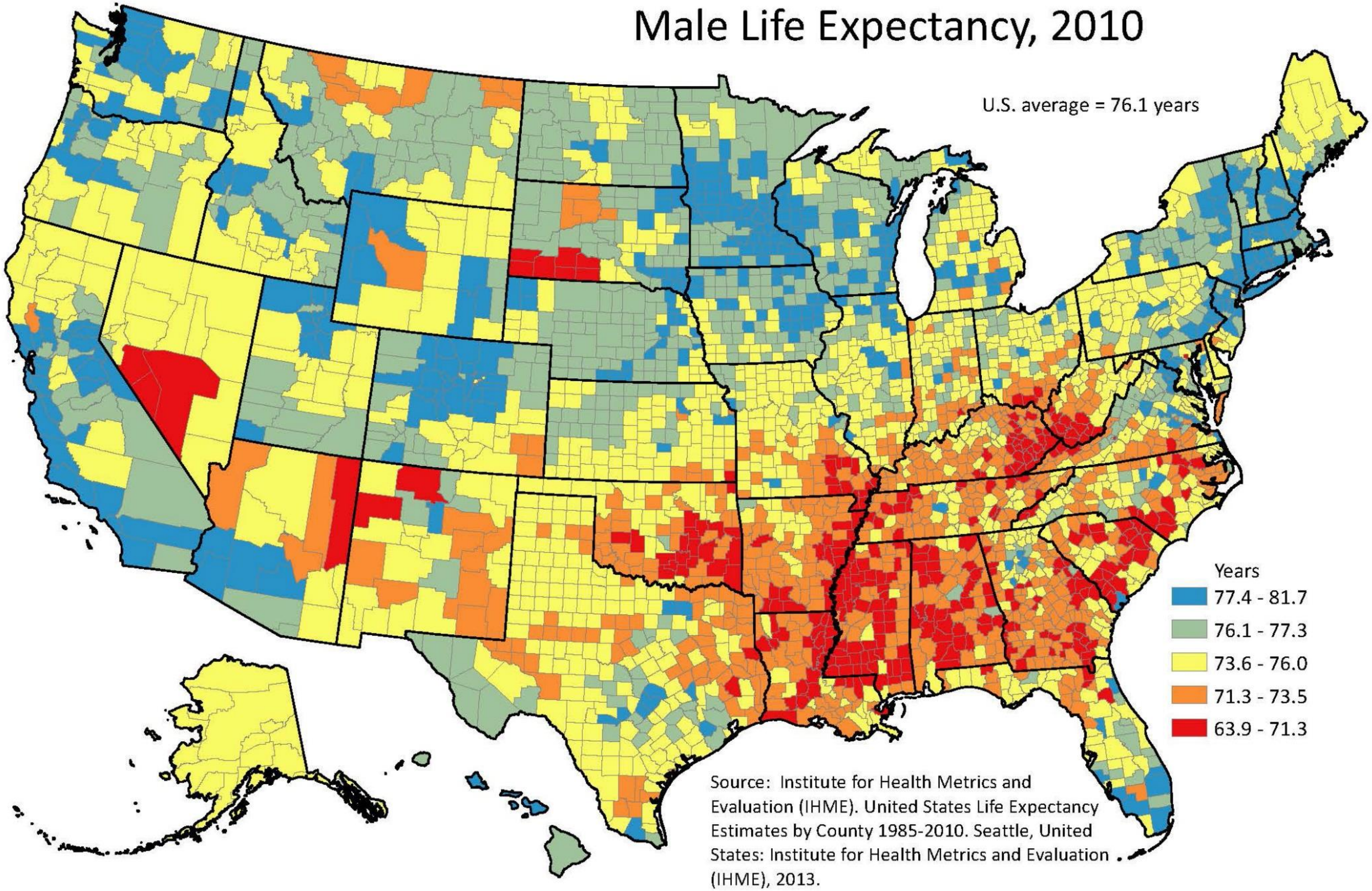
## Analysis of Variance

# Associations Between Variables

We are often interested in the relationship or "association" between two variables

How we measure that association in sample data—and how we infer how strong the association is in the population—depends on whether the variables are continuous or ordinal

What are the two variables depicted in the next three slides? Are the variables continuous or ordinal?

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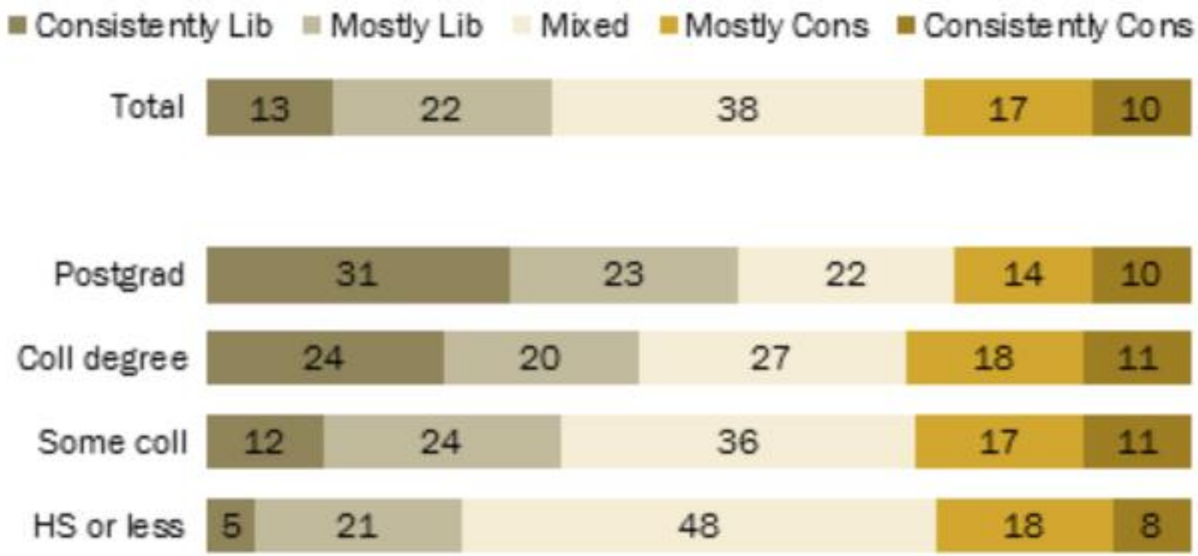
## Adults with postgraduate experience most likely to have consistently liberal political values

*% with political values that are...*



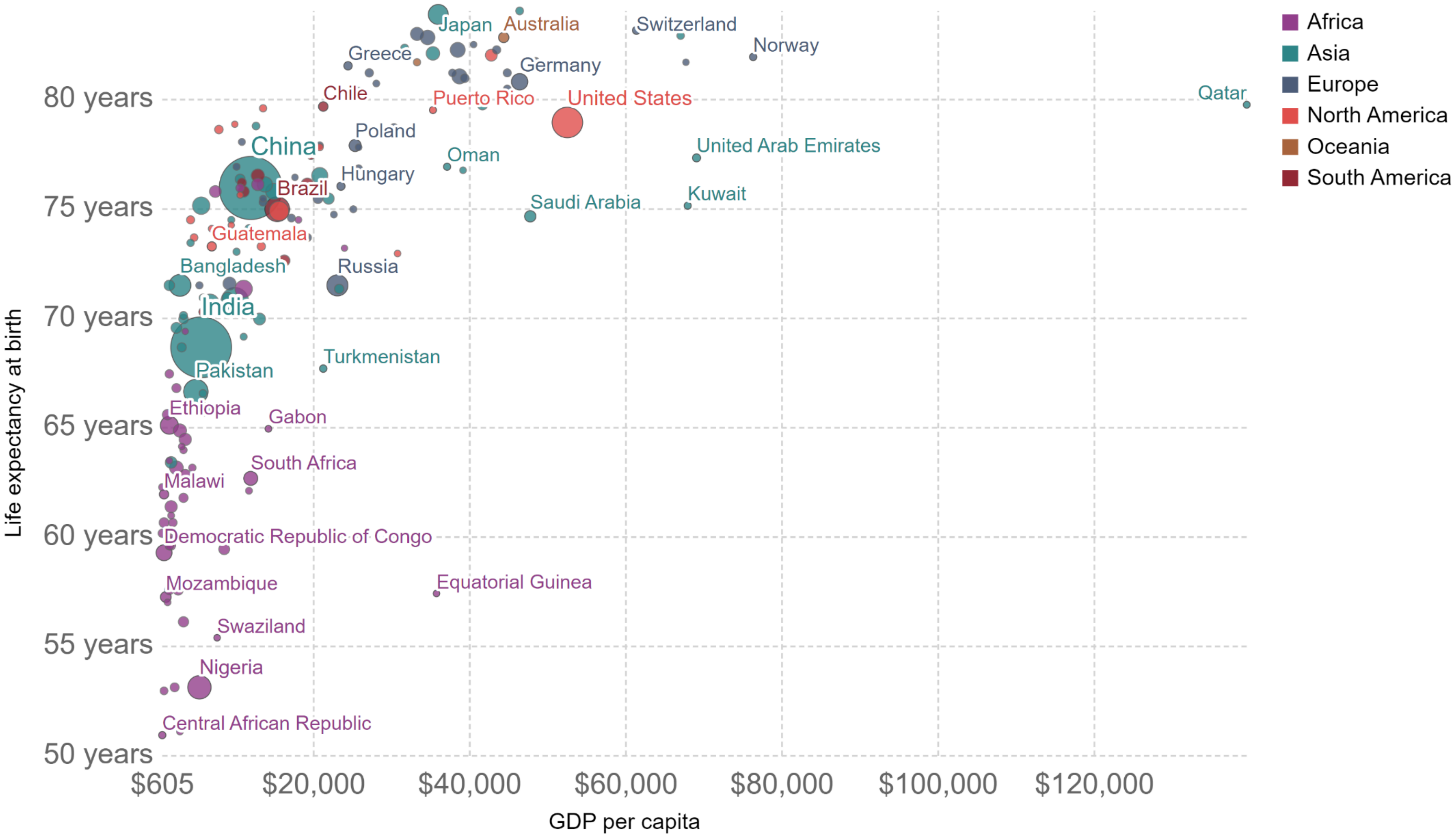

Source: Survey conducted Aug. 27-Oct. 4, 2015 (N=6,004). Ideological consistency based on a scale of 10 political values questions.

PEW RESEARCH CENTER

# Life expectancy vs. GDP per capita, 2015

Our World in Data

GDP per capita is measured in 2011 international dollars, which corrects for inflation and cross-country price differences.




Source: Clio-Infra & UN Population Division ; Maddison Project Database (2018)

# Associations Between Variables

Between now and the 3rd exam we will focus on measuring the association between two variables, X & Y

1. When X is <u>discrete</u> and Y is <u>continuous</u>, we will use "analysis of variance" techniques (Today)

2. When X and Y are <u>both discrete</u>, we will use cross-tabular and $\chi^2$ analyses (Thursday)

3. When X and Y are <u>both continuous</u>, we will use correlation & regression analyses (Next Week)

# ANALYSIS OF VARIANCE

# Introduction to ANOVA

**AN**alysis **O**f **VA**riance (**ANOVA**) techniques compare the mean of continuous variable Y across *J* populations

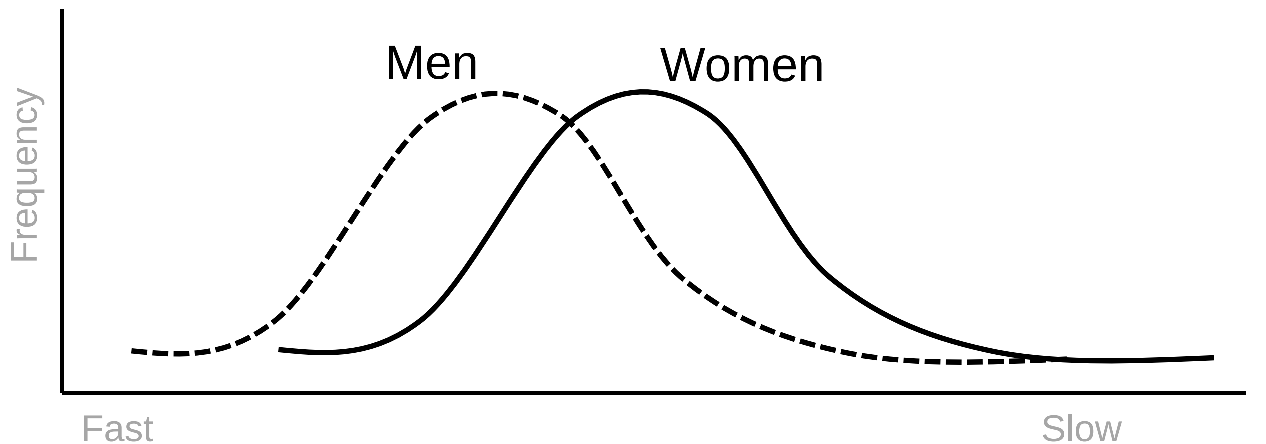
> Do people in difference counties have different life expectancies?
>
> Do people's political views (Y) vary by the type of community they live in (X … e.g., rural vs. urban vs. suburban)?
>
> Do people from different religious traditions (X) vary with respect to how much money they donate to political parties (Y)?

Such questions amount to comparing the mean of some continuous variable Y across the *J* categories of some discrete variable X

# Introduction to ANOVA

We saw something like this before…

How does the mean of continuous variable “race times” (Y) vary by levels of the categorical variable sex (X) ?

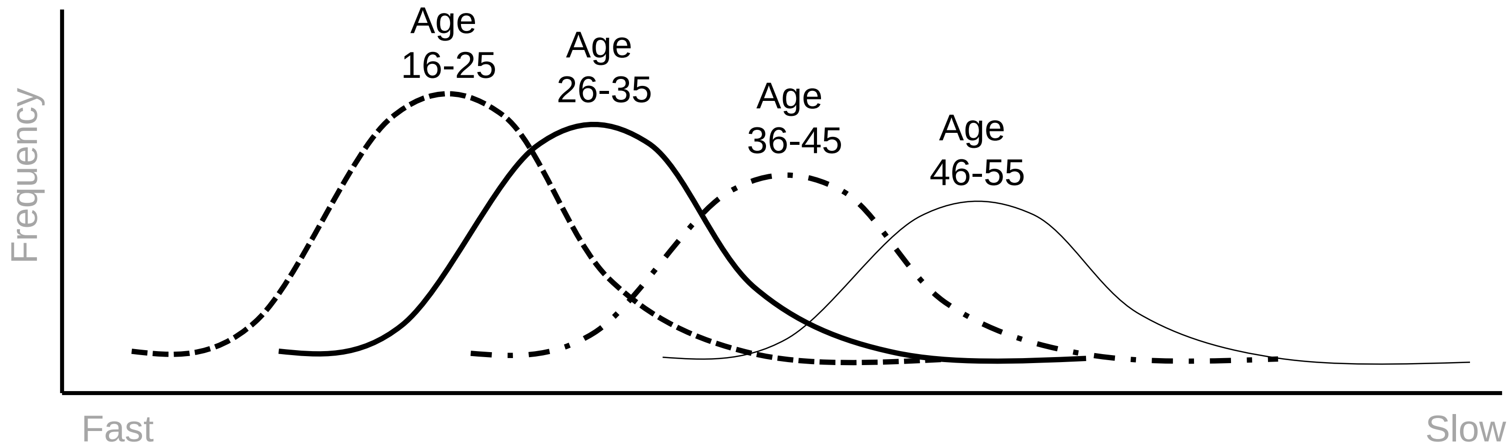



Using what we already know, we could test hypotheses or construct CI’s for the difference in means of race times between men and women

# Introduction to ANOVA

Now we want to allow for more categories of X

For example: How does the mean of continuous variable “race times” (Y) vary by levels of the categorical variable age (X) ?



# Introduction to ANOVA

ANOVA amounts to a test of the hypothesis that all of the *J* population means are equal:

$H_0$: $\mu_1=\mu_2=\mu_3=\ldots=\mu_J$

$H_1$: Not all of the means are equal

If we reject $H_0$, we do not know exactly which of the *J* means differs from the others, whether there are *J* different means, or what … we just know that not all of the *J* means are equal (Follow-up analyses may be necessary)

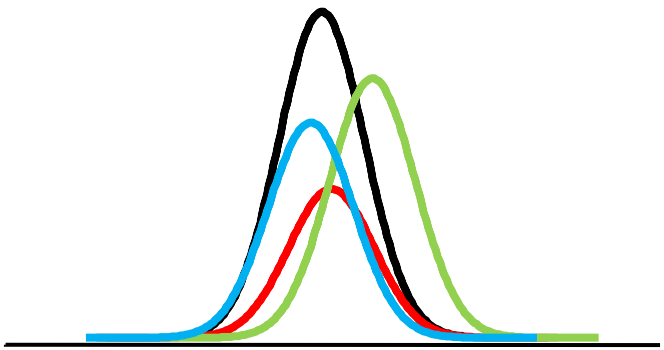
# Introduction to ANOVA

The test statistic that we use to perform the ANOVA hypothesis test is called the *F* statistic

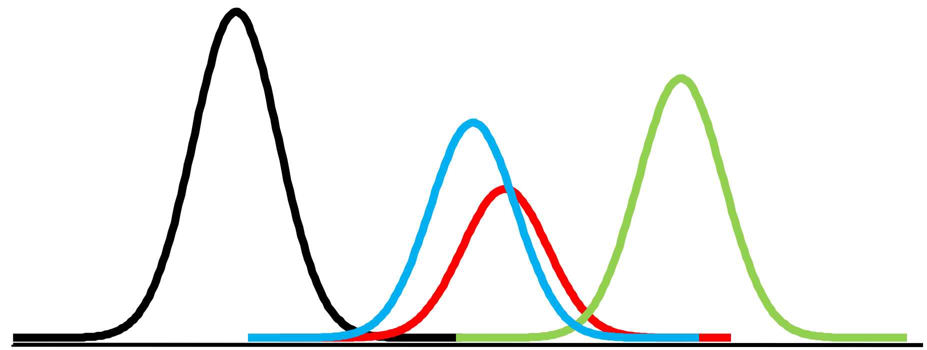
Conceptually, the more the *J* sample means differ, the larger the *F* statistic

If *F* is larger than we would expect by chance … if the observed value of the *F* test statistic exceeds a critical value determined in advance … then we reject $H_0$

# Introduction to ANOVA



Small *F* Value



Larger *F* Value

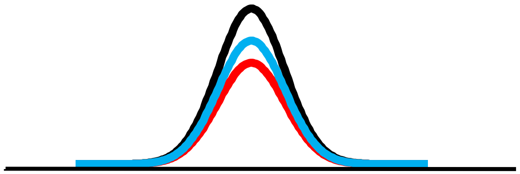
# The Logic of F Tests

The *F* statistic (defined later) is essentially a ratio:

$$F = \frac{\textit{Variation Between the j Sample Means}}{\textit{Variation Within Each of the j Groups}}$$

Imagine that we have *J*=3 groups & we compute 95% confidence intervals for the mean of Y within groups:

| | |
|---|---|
| Group #1 | 95% C.I. = $10 \pm 0.5$ |
| Group #2 | 95% C.I. = $10 \pm 0.5$ |
| Group #3 | 95% C.I. = $10 \pm 0.5$ |



Here, the *J* sample means do not vary between groups, but there is variability within groups. *F* would be tiny.

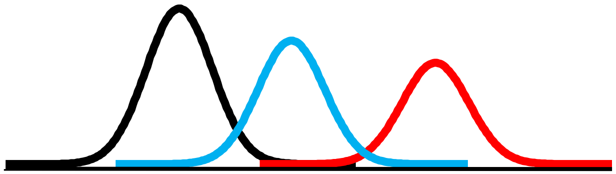
# The Logic of F Tests

The *F* statistic (defined later) is essentially a ratio:

$$F = \frac{\textit{Variation Between the j Sample Means}}{\textit{Variation Within Each of the j Groups}}$$

Now imagine that we have *J*=3 groups with these 95% confidence intervals:

| | |
|---|---|
| Group #1 | 95% C.I. = $5 \pm 0.1$ |
| Group #2 | 95% C.I. = $10 \pm 0.1$ |
| Group #3 | 95% C.I. = $15 \pm 0.1$ |



Here, the *J* sample means vary a lot between groups (and somewhat within groups). *F* would be large.

# The Logic of F Tests

Y varies from observation to observation

The *F* test amounts to asking how much of the variability in Y occurs between the *J* groups as opposed to within each of the *J* groups

Does which group you belong to—as indexed by the categorical variable X— “explain” or “account for” variability in Y?

# The Logic of F Tests

Another way to think about it: If $\mu$ is the overall mean of Y and $\mu_J$ is the mean for category J, then the "effect" of being in category J of the categorical variable X is $a_J$:

$$a_j = \mu_j - \mu$$

Rearranging this, the mean for category j equals

$$\mu_J = \mu + a_J$$

If the J group means are equal, then $\mu_J = \mu$ and $a_J = 0$ … implying that an individual's value of Y does not depend on the category of X to which they belong

# The Logic of F Tests

For individual i we can express the value of Y as:

$$Y_{ij} = \mu + a_j + e_{ij}$$

where $Y_{ij}$ is the value of Y for individual i in group j, $\mu$ is the overall mean of Y, $a_j$ is the effect of being in category j of variable X, and $e_{ij}$ is a random error term

ANOVA is an effort to determine how much of the variance in $Y_{ij}$ is attributable to group membership (the $a_j$) and how much is due to other things ($e_{ij}$)

How do we know how much of the variation in $Y_{ij}$ is due to these two source of variability?

# The Logic of F Tests

The core idea of ANOVA is that we can separate the variance in Y into two components

1. Variability between groups
2. Variability within groups

The *F*-statistic is based on a comparison of between-group variability to within-group variability

How do we measure between-group variability?

How do we measure within-group variability?

# The Logic of F Tests

Return to the formula for sample variance:

$$s_Y^2 = \frac{\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)^2}{N-1}$$

If we then subscript Y with "i" to represent each individual and "j" to represent the category of X to which they belong, then we can re-write the numerator of the formula for sample variance as

$$\sum_{i=1}^{N}\left(Y_i - \overline{Y}\right)^2 = \sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij} - \overline{Y})^2$$

# The Logic of F Tests

The quantity

$$\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\overline{Y})^2$$

is called the "total sum of squares," or $SS_{TOTAL}$

This quantity can be decomposed into two parts:

$$\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\overline{Y})^2=\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\overline{Y}_j)^2+\sum_{j=1}^{J}n_j(\overline{Y}_j-\overline{Y})^2$$

# The Logic of F Tests

$$\underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\bar{Y})^2}_{SS_{TOTAL}} = \underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\bar{Y}_j)^2}_{SS_{WITHIN}} + \underbrace{\sum_{j=1}^{J}n_j(\bar{Y}_j-\bar{Y})^2}_{SS_{BETWEEN}}$$

$SS_{TOTAL}$ = Sum of squared deviations of each individual's value of Y from the overall sample mean of Y

$SS_{WITHIN}$ = Sum of the squared deviations of each individual's value of Y from their group's mean of Y

$SS_{BETWEEN}$ = Sum of the squared deviations of each group's mean value of Y from the overall sample mean of Y

# The F Statistic

The F statistic is defined as

$$F_{J-1,N-J} = \frac{SS_{BETWEEN}/J-1}{SS_{WITHIN}/N-J} = \frac{MS_{BETWEEN}}{MS_{WITHIN}}$$

where

$$MS_{BETWEEN} = \frac{SS_{BETWEEN}}{J-1} \qquad MS_{WITHIN} = \frac{SS_{WITHIN}}{N-J}$$

$MS_{BETWEEN}$ is an estimate of the amount of variance in Y attributable to the category of X to which cases belong

$MS_{WITHIN}$ is an estimate of the amount of variance in Y attributable to everything else

# The F Statistic

The F statistic is defined as

$$F_{J-1,N-J} = \frac{SS_{BETWEEN}/J-1}{SS_{WITHIN}/N-J} = \frac{MS_{BETWEEN}}{MS_{WITHIN}}$$

where

$$MS_{BETWEEN} = \frac{SS_{BETWEEN}}{J-1} \qquad MS_{WITHIN} = \frac{SS_{WITHIN}}{N-J}$$

If the mean of Y does not vary across groups, then the ratio F will be small

Hypothesis test: Is the value of F larger than we would expect by chance if $\mu_1=\mu_2=...=\mu_j$?

# The F Statistic

The F statistic is defined as

$$F_{J-1,N-J} = \frac{SS_{BETWEEN}/J-1}{SS_{WITHIN}/N-J} = \frac{MS_{BETWEEN}}{MS_{WITHIN}}$$

where

$$MS_{BETWEEN} = \frac{SS_{BETWEEN}}{J-1} \qquad MS_{WITHIN} = \frac{SS_{WITHIN}}{N-J}$$

$MS_{BETWEEN}$ has j — 1 degrees of freedom

$MS_{WITHIN}$ has N — j degrees of freedom

F thus has two degrees of freedom (df): The "numerator" $df_{NUM}$ ($\nu_1$=j—1) and the "denominator" $df_{DENOM}$ ($\nu_2$=N—j)

# The F Statistic

-> group = 1

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 50,000 | 100.0316 | 10.04977 | 51 | 145 |

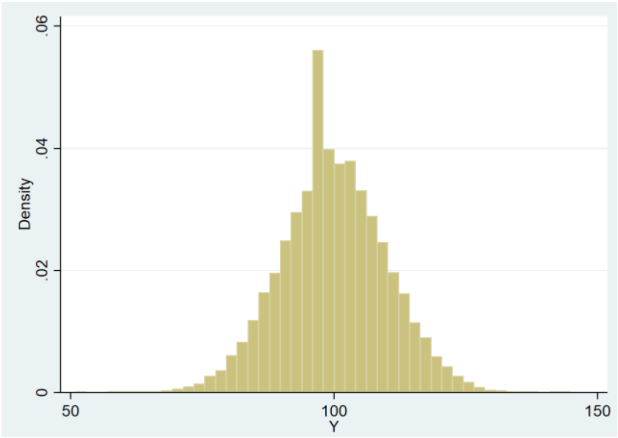
-> group = 2

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 50,000 | 100.0316 | 10.04977 | 51 | 145 |

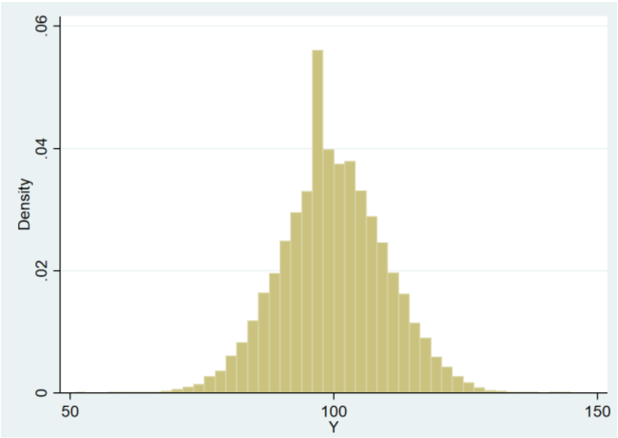
-> group = 3

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 50,000 | 100.0316 | 10.04977 | 51 | 145 |

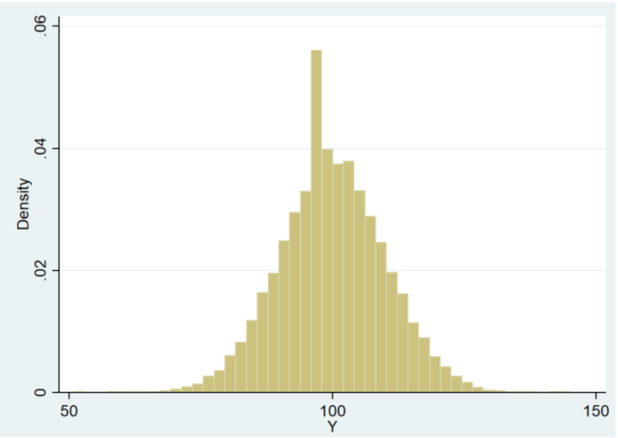
# The F Statistic




Group 1




Group 2




Group 3

# The F Statistic

The F statistic is defined as

$$F_{J-1,N-J} = \frac{SS_{BETWEEN}/J-1}{SS_{WITHIN}/N-J} = \frac{\mathbf{0}/2}{15{,}149{,}371/149{,}997} = \mathbf{0}$$

$$\underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\bar{Y})^2}_{SS_{TOTAL}} = \underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\bar{Y}_j)^2}_{SS_{WITHIN}} + \underbrace{\sum_{j=1}^{J}n_j(\bar{Y}_j-\bar{Y})^2}_{SS_{BETWEEN}}$$

# The F Statistic

```
-> group = 1
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 20 | 103.05 | 10.33886 | 81 | 118 |

```
-> group = 2
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 20 | 99.9 | 11.15866 | 80 | 117 |

```
-> group = 3
```

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 20 | 101.15 | 9.911583 | 79 | 116 |

# The F Statistic

The F statistic is for this random sample of 60 (20 from each of the three groups) is:

$$F_{J-1,N-J} = \frac{SS_{BETWEEN}/J-1}{SS_{WITHIN}/N-J} = \frac{100.63/2}{6{,}263.3/149{,}997} = \mathbf{0.46}$$

$$\underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\overline{Y})^2}_{SS_{TOTAL}} = \underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\overline{Y}_j)^2}_{SS_{WITHIN}} + \underbrace{\sum_{j=1}^{J}n_j(\overline{Y}_j-\overline{Y})^2}_{SS_{BETWEEN}}$$

$SS_{TOTAL}$ = $SS_{WITHIN}$ + $SS_{BETWEEN}$

# The F Statistic

-> group = 1

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 20 | 95.2 | 9.311001 | 77 | 112 |

-> group = 2

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 20 | 103.7 | 11.59446 | 77 | 124 |

-> group = 3

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| Y | 20 | 97.9 | 11.83616 | 72 | 116 |

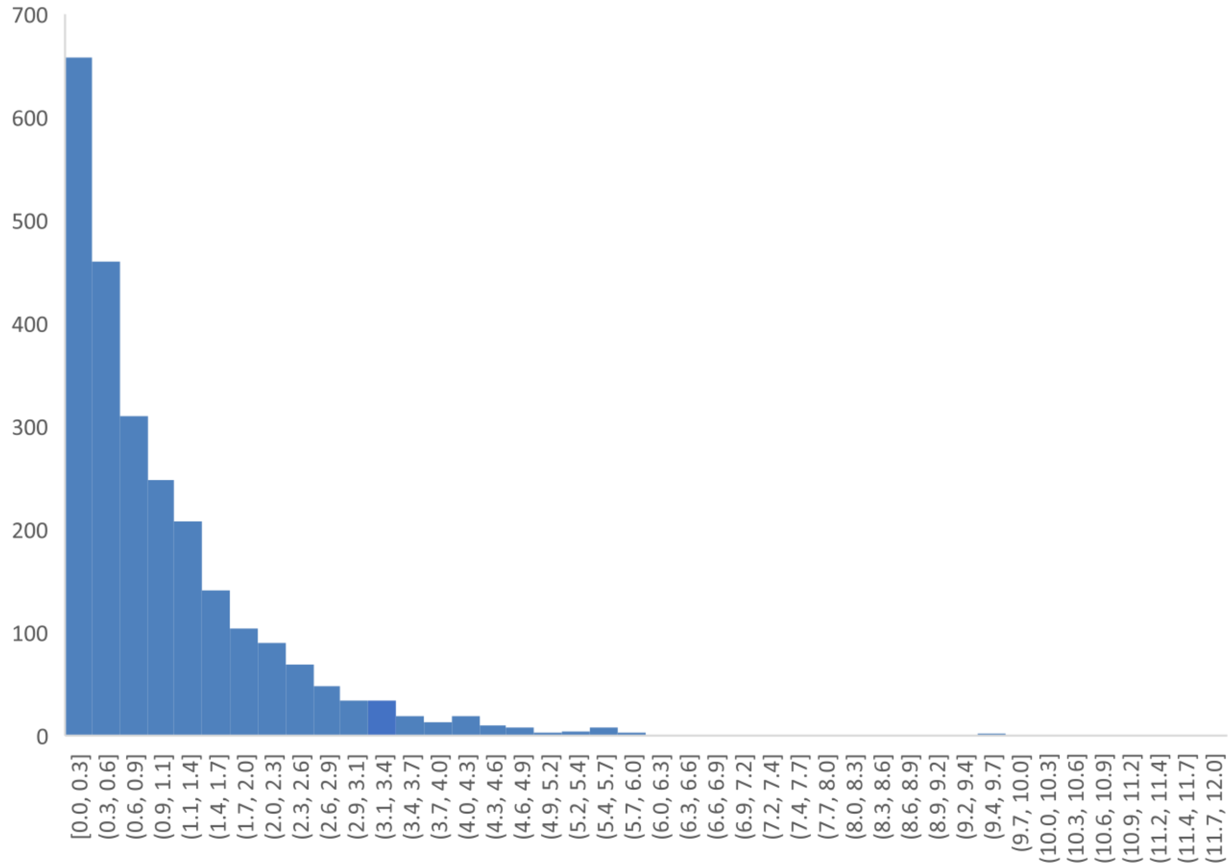
# The F Statistic

The F statistic is for this random sample of 60 (20 from each of the three groups) is:

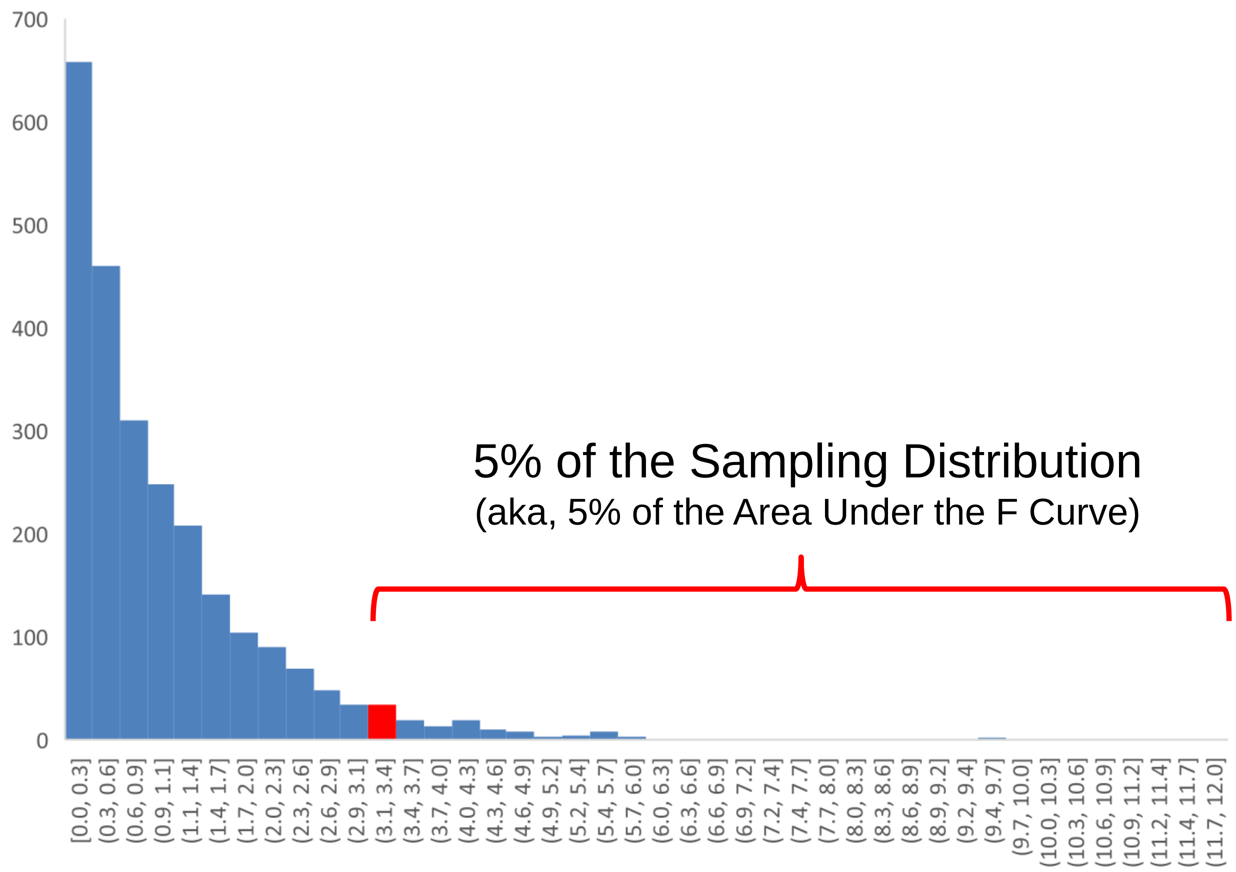
$$F_{J-1,N-J} = \frac{SS_{BETWEEN}/J-1}{SS_{WITHIN}/N-J} = \frac{754.53/2}{6,863/149,997} = \mathbf{3.13}$$

$$\underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\bar{Y})^2}_{SS_{TOTAL}} = \underbrace{\sum_{j=1}^{J}\sum_{i=1}^{n_j}(Y_{ij}-\bar{Y}_j)^2}_{SS_{WITHIN}} + \underbrace{\sum_{j=1}^{J}n_j(\bar{Y}_j-\bar{Y})^2}_{SS_{BETWEEN}}$$

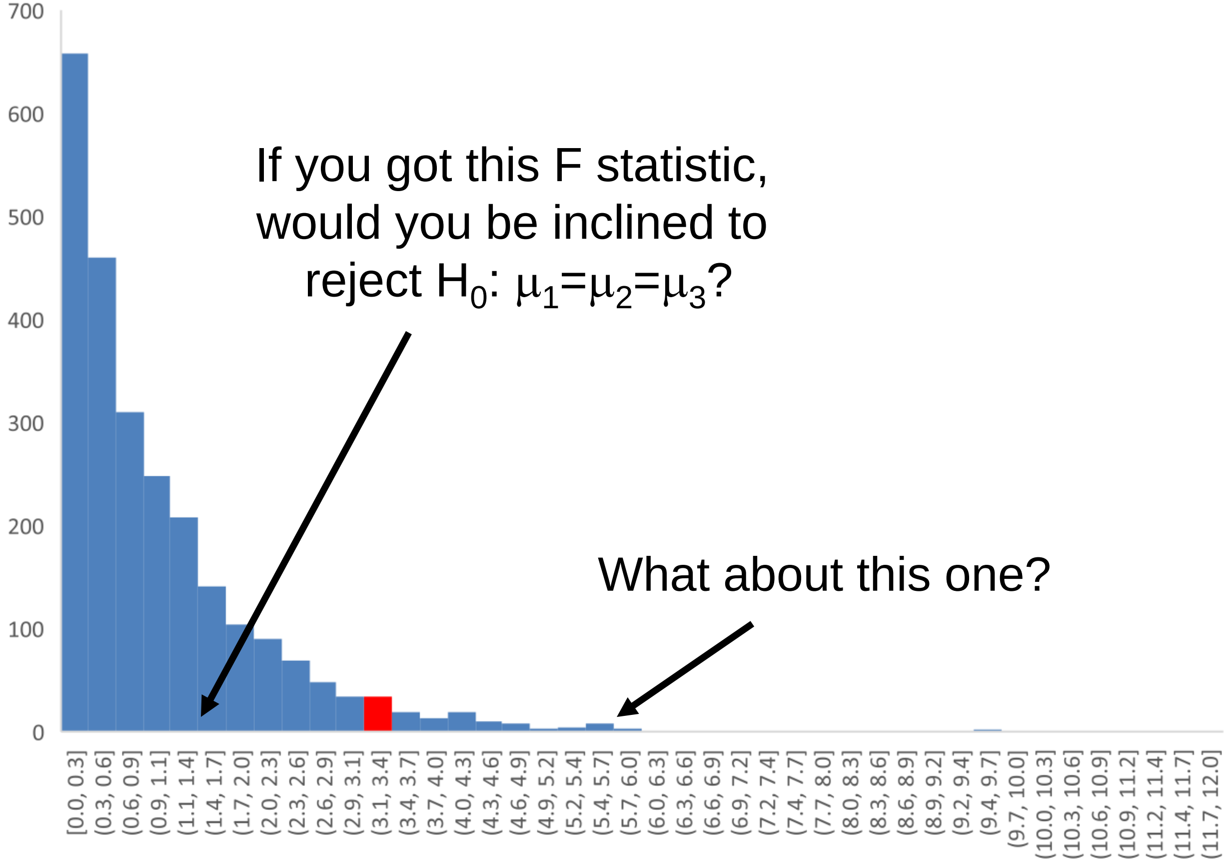
# The F Statistic



# The F Statistic



# The F Statistic



# ANOVA Example

Say that Y is people's political views (where 1=Extremely Liberal and 7=Extremely Conservative) and X represents the type of community that people live in; here, J=5

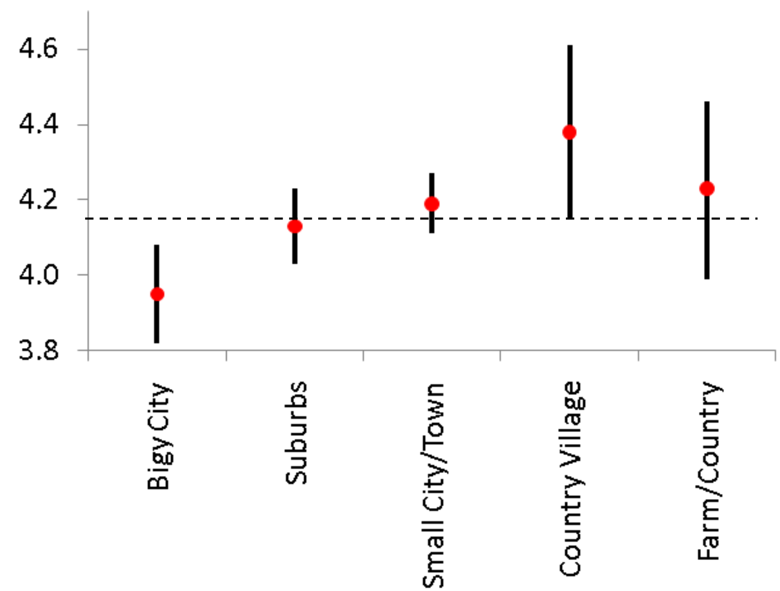
N for the full sample equals 2,779 with an overall sample mean of Y equal to 4.15

| | | | |
|---|---|---|---|
| Big City | Mean=3.95 | n=522 | (95% C.I. = 3.82 to 4.08) |
| Suburbs | Mean=4.13 | n=724 | (95% C.I. = 4.03 to 4.23) |
| Small City/Town | Mean=4.19 | n=1,093 | (95% C.I. = 4.11 to 4.27) |
| Country Village | Mean=4.38 | n=122 | (95% C.I. = 4.15 to 4.61) |
| Farm/Country | Mean=4.30 | n=318 | (95% C.I. = 3.99 to 4.46) |

# ANOVA Example

Say that Y is people's political views (where 1=Extremely Liberal and 7=Extremely Conservative) and X represents the type of community that people live in; here, J=5

N for the full sample equals 2,779 with an overall sample mean of Y equal to 4.15



# ANOVA Example

Hypothesis Testing in 6 Steps … Just Like Before

1. State the null ($H_0$) and alternative ($H_1$) hypotheses
2. Check that the sample data conform to basic assumptions; if they do not, then do not go any further
3. Choose an $\alpha$ probability level … that is, a probability associated with incorrectly rejecting the null hypothesis
4. Determine the "critical value" … that is, how large the test statistic must be in order to reject the null hypothesis at the given $\alpha$ level
5. Calculate the test statistic ... F
6. Compare the test statistic to the critical value

# ANOVA Example

State the null ($H_0$) and alternative ($H_1$) hypotheses

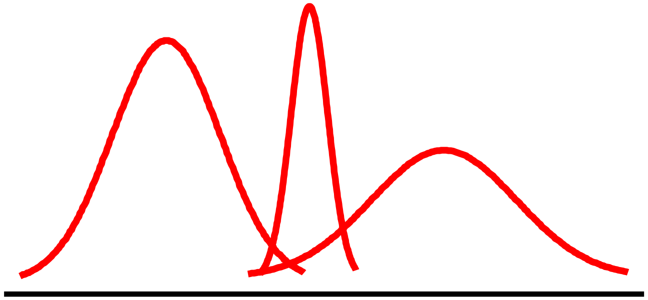
$H_0$: $\mu_{\text{Big City}} = \mu_{\text{Suburbs}} = \mu_{\text{Small City}} = \mu_{\text{Village}} = \mu_{\text{Farm}}$

$H_1$: Not all of the population group means are equal

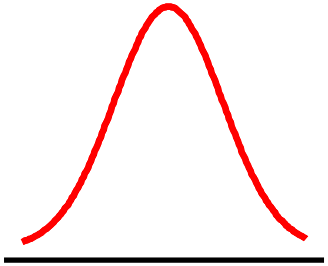
# ANOVA Example

Check that the sample data conform to basic assumptions; if they do not, then do not go any further

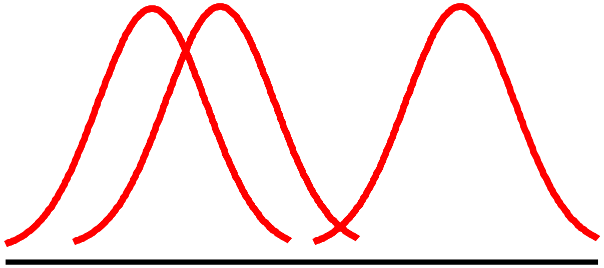
1. The *j* samples are independent random samples
2. Within each population group, Y is normally distributed
3. The standard deviation of Y is equal across the *j* population groups (the “homoskedasticity” assumption)



**Assumptions Are Not Met**



**$H_0$: $\mu_1=\mu_2=\mu_3$**



**$H_1$: Not all Means Are Equal**

# ANOVA Example

Choose an $\alpha$ probability level … that is, a probability associated with incorrectly rejecting the null hypothesis

Let's choose $\alpha$=0.05

Determine the "critical value" … that is, how large the test statistic must be in order to reject the null hypothesis at the given $\alpha$ level

As shown in an F table, the critical value of F depends on $\alpha$, $df_{NUM}$ (which equals J-1), and $df_{DENOM}$ (which equals N-J)

# Critical Values of F
## (α=0.05)

| DENOMINATOR Degrees of Freedom | NUMERATOR Degrees of Freedom | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **15** | **20** | **30** | **40** | **50** | **100** | **200** | **∞** |
| **1** | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 245.95 | 248.01 | 250.10 | 251.14 | 251.77 | 253.04 | 253.68 | 254.31 |
| **2** | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.43 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.49 | 19.50 |
| **3** | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.70 | 8.66 | 8.62 | 8.59 | 8.58 | 8.55 | 8.54 | 8.53 |
| **4** | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.86 | 5.80 | 5.75 | 5.72 | 5.70 | 5.66 | 5.65 | 5.63 |
| **5** | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.62 | 4.56 | 4.50 | 4.46 | 4.44 | 4.41 | 4.39 | 4.36 |
| **6** | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 3.94 | 3.87 | 3.81 | 3.77 | 3.75 | 3.71 | 3.69 | 3.67 |
| **7** | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.51 | 3.44 | 3.38 | 3.34 | 3.32 | 3.27 | 3.25 | 3.23 |
| **8** | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.22 | 3.15 | 3.08 | 3.04 | 3.02 | 2.97 | 2.95 | 2.93 |
| **9** | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.01 | 2.94 | 2.86 | 2.83 | 2.80 | 2.76 | 2.73 | 2.71 |
| **10** | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.85 | 2.77 | 2.70 | 2.66 | 2.64 | 2.59 | 2.56 | 2.54 |
| **11** | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.72 | 2.65 | 2.57 | 2.53 | 2.51 | 2.46 | 2.43 | 2.40 |
| **12** | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.62 | 2.54 | 2.47 | 2.43 | 2.40 | 2.35 | 2.32 | 2.30 |
| **13** | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.53 | 2.46 | 2.38 | 2.34 | 2.31 | 2.26 | 2.23 | 2.21 |
| **14** | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.46 | 2.39 | 2.31 | 2.27 | 2.24 | 2.19 | 2.16 | 2.13 |
| **15** | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.40 | 2.33 | 2.25 | 2.20 | 2.18 | 2.12 | 2.10 | 2.07 |
| **16** | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.35 | 2.28 | 2.19 | 2.15 | 2.12 | 2.07 | 2.04 | 2.01 |
| **17** | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.31 | 2.23 | 2.15 | 2.10 | 2.08 | 2.02 | 1.99 | 1.96 |
| **18** | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.27 | 2.19 | 2.11 | 2.06 | 2.04 | 1.98 | 1.95 | 1.92 |
| **19** | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.23 | 2.16 | 2.07 | 2.03 | 2.00 | 1.94 | 1.91 | 1.88 |
| **20** | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.20 | 2.12 | 2.04 | 1.99 | 1.97 | 1.91 | 1.88 | 1.84 |
| **21** | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.18 | 2.10 | 2.01 | 1.96 | 1.94 | 1.88 | 1.84 | 1.81 |
| **22** | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.15 | 2.07 | 1.98 | 1.94 | 1.91 | 1.85 | 1.82 | 1.78 |
| **23** | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.13 | 2.05 | 1.96 | 1.91 | 1.88 | 1.82 | 1.79 | 1.76 |
| **24** | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.11 | 2.03 | 1.94 | 1.89 | 1.86 | 1.80 | 1.77 | 1.73 |
| **25** | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.09 | 2.01 | 1.92 | 1.87 | 1.84 | 1.78 | 1.75 | 1.71 |
| **26** | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.07 | 1.99 | 1.90 | 1.85 | 1.82 | 1.76 | 1.73 | 1.69 |
| **27** | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.06 | 1.97 | 1.88 | 1.84 | 1.81 | 1.74 | 1.71 | 1.67 |
| **28** | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.04 | 1.96 | 1.87 | 1.82 | 1.79 | 1.73 | 1.69 | 1.65 |
| **29** | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.03 | 1.94 | 1.85 | 1.81 | 1.77 | 1.71 | 1.67 | 1.64 |
| **30** | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.01 | 1.93 | 1.84 | 1.79 | 1.76 | 1.70 | 1.66 | 1.62 |
| **31** | 4.16 | 3.30 | 2.91 | 2.68 | 2.52 | 2.41 | 2.32 | 2.25 | 2.20 | 2.15 | 2.00 | 1.92 | 1.83 | 1.78 | 1.75 | 1.68 | 1.65 | 1.61 |
| **32** | 4.15 | 3.29 | 2.90 | 2.67 | 2.51 | 2.40 | 2.31 | 2.24 | 2.19 | 2.14 | 1.99 | 1.91 | 1.82 | 1.77 | 1.74 | 1.67 | 1.63 | 1.59 |
| **33** | 4.14 | 3.28 | 2.89 | 2.66 | 2.50 | 2.39 | 2.30 | 2.23 | 2.18 | 2.13 | 1.98 | 1.90 | 1.81 | 1.76 | 1.72 | 1.66 | 1.62 | 1.58 |
| **34** | 4.13 | 3.28 | 2.88 | 2.65 | 2.49 | 2.38 | 2.29 | 2.23 | 2.17 | 2.12 | 1.97 | 1.89 | 1.80 | 1.75 | 1.71 | 1.65 | 1.61 | 1.57 |
| **35** | 4.12 | 3.27 | 2.87 | 2.64 | 2.49 | 2.37 | 2.29 | 2.22 | 2.16 | 2.11 | 1.96 | 1.88 | 1.79 | 1.74 | 1.70 | 1.63 | 1.60 | 1.56 |
| **40** | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 1.92 | 1.84 | 1.74 | 1.69 | 1.66 | 1.59 | 1.55 | 1.51 |
| **50** | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.87 | 1.78 | 1.69 | 1.63 | 1.60 | 1.52 | 1.48 | 1.44 |
| **75** | 3.97 | 3.12 | 2.73 | 2.49 | 2.34 | 2.22 | 2.13 | 2.06 | 2.01 | 1.96 | 1.80 | 1.71 | 1.61 | 1.55 | 1.52 | 1.44 | 1.39 | 1.34 |
| **100** | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.77 | 1.68 | 1.57 | 1.52 | 1.48 | 1.39 | 1.34 | 1.28 |
| **200** | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.72 | 1.62 | 1.52 | 1.46 | 1.41 | 1.32 | 1.26 | 1.19 |
| **∞** | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.67 | 1.57 | 1.46 | 1.39 | 1.35 | 1.24 | 1.17 | 1.00 |

# Worksheet

With $\alpha$=0.05, what is the critical value of *F* when…

$df_{NUM}$=3, and $df_{DENOM}$=120 ?

$df_{NUM}$=10, and $df_{DENOM}$=20 ?

$df_{NUM}$=20, and $df_{DENOM}$=30 ?

With $\alpha$=0.01, what is the critical value of *F* when…

$df_{NUM}$=1, and $df_{DENOM}$=1,000 ?

$df_{NUM}$=4, and $df_{DENOM}$=20 ?

# Critical Values of F
## ($\alpha$=0.05)

| DENOMINATOR Degrees of Freedom \ NUMERATOR Degrees of Freedom | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 30 | 40 | 50 | 100 | 200 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 245.95 | 248.01 | 250.10 | 251.14 | 251.77 | 253.04 | 253.68 | 254.31 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.43 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.49 | 19.50 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.70 | 8.66 | 8.62 | 8.59 | 8.58 | 8.55 | 8.54 | 8.53 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.86 | 5.80 | 5.75 | 5.72 | 5.70 | 5.66 | 5.65 | 5.63 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.62 | 4.56 | 4.50 | 4.46 | 4.44 | 4.41 | 4.39 | 4.36 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 3.94 | 3.87 | 3.81 | 3.77 | 3.75 | 3.71 | 3.69 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.51 | 3.44 | 3.38 | 3.34 | 3.32 | 3.27 | 3.25 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.22 | 3.15 | 3.08 | 3.04 | 3.02 | 2.97 | 2.95 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.01 | 2.94 | 2.86 | 2.83 | 2.80 | 2.76 | 2.73 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.85 | 2.77 | 2.70 | 2.66 | 2.64 | 2.59 | 2.56 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.72 | 2.65 | 2.57 | 2.53 | 2.51 | 2.46 | 2.43 | 2.40 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.62 | 2.54 | 2.47 | 2.43 | 2.40 | 2.35 | 2.32 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.53 | 2.46 | 2.38 | 2.34 | 2.31 | 2.26 | 2.23 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.46 | 2.39 | 2.31 | 2.27 | 2.24 | 2.19 | 2.16 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.40 | 2.33 | 2.25 | 2.20 | 2.18 | 2.12 | 2.10 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.35 | 2.28 | 2.19 | 2.15 | 2.12 | 2.07 | 2.04 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.31 | 2.23 | 2.15 | 2.10 | 2.08 | 2.02 | 1.99 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.27 | 2.19 | 2.11 | 2.06 | 2.04 | 1.98 | 1.95 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.23 | 2.16 | 2.07 | 2.03 | 2.00 | 1.94 | 1.91 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.20 | 2.12 | 2.04 | 1.99 | 1.97 | 1.91 | 1.88 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.18 | 2.10 | 2.01 | 1.96 | 1.94 | 1.88 | 1.84 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.15 | 2.07 | 1.98 | 1.94 | 1.91 | 1.85 | 1.82 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.13 | 2.05 | 1.96 | 1.91 | 1.88 | 1.82 | 1.79 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.11 | 2.03 | 1.94 | 1.89 | 1.86 | 1.80 | 1.77 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.09 | 2.01 | 1.92 | 1.87 | 1.84 | 1.78 | 1.75 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.07 | 1.99 | 1.90 | 1.85 | 1.82 | 1.76 | 1.73 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.06 | 1.97 | 1.88 | 1.84 | 1.81 | 1.74 | 1.71 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.04 | 1.96 | 1.87 | 1.82 | 1.79 | 1.73 | 1.69 | 1.65 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.03 | 1.94 | 1.85 | 1.81 | 1.77 | 1.71 | 1.67 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.01 | 1.93 | 1.84 | 1.79 | 1.76 | 1.70 | 1.66 | 1.62 |
| 31 | 4.16 | 3.30 | 2.91 | 2.68 | 2.52 | 2.41 | 2.32 | 2.25 | 2.20 | 2.15 | 2.00 | 1.92 | 1.83 | 1.78 | 1.75 | 1.68 | 1.65 | 1.61 |
| 32 | 4.15 | 3.29 | 2.90 | 2.67 | 2.51 | 2.40 | 2.31 | 2.24 | 2.19 | 2.14 | 1.99 | 1.91 | 1.82 | 1.77 | 1.74 | 1.67 | 1.63 | 1.59 |
| 33 | 4.14 | 3.28 | 2.89 | 2.66 | 2.50 | 2.39 | 2.30 | 2.23 | 2.18 | 2.13 | 1.98 | 1.90 | 1.81 | 1.76 | 1.72 | 1.66 | 1.62 | 1.58 |
| 34 | 4.13 | 3.28 | 2.88 | 2.65 | 2.49 | 2.38 | 2.29 | 2.23 | 2.17 | 2.12 | 1.97 | 1.89 | 1.80 | 1.75 | 1.71 | 1.65 | 1.61 | 1.57 |
| 35 | 4.12 | 3.27 | 2.87 | 2.64 | 2.49 | 2.37 | 2.29 | 2.22 | 2.16 | 2.11 | 1.96 | 1.88 | 1.79 | 1.74 | 1.70 | 1.63 | 1.60 | 1.56 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 1.92 | 1.84 | 1.74 | 1.69 | 1.66 | 1.59 | 1.55 | 1.51 |
| 50 | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.87 | 1.78 | 1.69 | 1.63 | 1.60 | 1.52 | 1.48 | 1.44 |
| 75 | 3.97 | 3.12 | 2.73 | 2.49 | 2.34 | 2.22 | 2.13 | 2.06 | 2.01 | 1.96 | 1.80 | 1.71 | 1.61 | 1.55 | 1.52 | 1.44 | 1.39 | 1.34 |
| 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.77 | 1.68 | 1.57 | 1.52 | 1.48 | 1.39 | 1.34 | 1.28 |
| 200 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.72 | 1.62 | 1.52 | 1.46 | 1.41 | 1.32 | 1.26 | 1.19 |
| ∞ | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.67 | 1.57 | 1.46 | 1.39 | 1.35 | 1.24 | 1.17 | 1.00 |

# Critical Values of F
## ($\alpha$=0.01)

| DENOMINATOR Degrees of Freedom | NUMERATOR Degrees of Freedom | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 30 | 40 | 50 | 100 | 200 | ∞ |
| 1 | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 | 6055.85 | 6157.28 | 6208.73 | 6260.65 | 6286.78 | 6302.52 | 6334.11 | 6349.97 | 6365.86 |
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.43 | 99.45 | 99.47 | 99.47 | 99.48 | 99.49 | 99.49 | 99.50 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 26.87 | 26.69 | 26.50 | 26.41 | 26.35 | 26.24 | 26.18 | 26.13 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.20 | 14.02 | 13.84 | 13.75 | 13.69 | 13.58 | 13.52 | 13.46 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.72 | 9.55 | 9.38 | 9.29 | 9.24 | 9.13 | 9.08 | 9.02 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.56 | 7.40 | 7.23 | 7.14 | 7.09 | 6.99 | 6.93 | 6.88 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.31 | 6.16 | 5.99 | 5.91 | 5.86 | 5.75 | 5.70 | 5.65 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.52 | 5.36 | 5.20 | 5.12 | 5.07 | 4.96 | 4.91 | 4.86 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 4.96 | 4.81 | 4.65 | 4.57 | 4.52 | 4.41 | 4.36 | 4.31 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.56 | 4.41 | 4.25 | 4.17 | 4.12 | 4.01 | 3.96 | 3.91 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.25 | 4.10 | 3.94 | 3.86 | 3.81 | 3.71 | 3.66 | 3.60 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.01 | 3.86 | 3.70 | 3.62 | 3.57 | 3.47 | 3.41 | 3.36 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.82 | 3.66 | 3.51 | 3.43 | 3.38 | 3.27 | 3.22 | 3.17 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.66 | 3.51 | 3.35 | 3.27 | 3.22 | 3.11 | 3.06 | 3.00 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.52 | 3.37 | 3.21 | 3.13 | 3.08 | 2.98 | 2.92 | 2.87 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.41 | 3.26 | 3.10 | 3.02 | 2.97 | 2.86 | 2.81 | 2.75 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.31 | 3.16 | 3.00 | 2.92 | 2.87 | 2.76 | 2.71 | 2.65 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.23 | 3.08 | 2.92 | 2.84 | 2.78 | 2.68 | 2.62 | 2.57 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.15 | 3.00 | 2.84 | 2.76 | 2.71 | 2.60 | 2.55 | 2.49 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.09 | 2.94 | 2.78 | 2.69 | 2.64 | 2.54 | 2.48 | 2.42 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.03 | 2.88 | 2.72 | 2.64 | 2.58 | 2.48 | 2.42 | 2.36 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 2.98 | 2.83 | 2.67 | 2.58 | 2.53 | 2.42 | 2.36 | 2.31 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 2.93 | 2.78 | 2.62 | 2.54 | 2.48 | 2.37 | 2.32 | 2.26 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 2.89 | 2.74 | 2.58 | 2.49 | 2.44 | 2.33 | 2.27 | 2.21 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.85 | 2.70 | 2.54 | 2.45 | 2.40 | 2.29 | 2.23 | 2.17 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.81 | 2.66 | 2.50 | 2.42 | 2.36 | 2.25 | 2.19 | 2.13 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.78 | 2.63 | 2.47 | 2.38 | 2.33 | 2.22 | 2.16 | 2.10 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.75 | 2.60 | 2.44 | 2.35 | 2.30 | 2.19 | 2.13 | 2.06 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.73 | 2.57 | 2.41 | 2.33 | 2.27 | 2.16 | 2.10 | 2.03 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.70 | 2.55 | 2.39 | 2.30 | 2.25 | 2.13 | 2.07 | 2.01 |
| 31 | 7.53 | 5.36 | 4.48 | 3.99 | 3.67 | 3.45 | 3.28 | 3.15 | 3.04 | 2.96 | 2.68 | 2.52 | 2.36 | 2.27 | 2.22 | 2.11 | 2.04 | 1.98 |
| 32 | 7.50 | 5.34 | 4.46 | 3.97 | 3.65 | 3.43 | 3.26 | 3.13 | 3.02 | 2.93 | 2.65 | 2.50 | 2.34 | 2.25 | 2.20 | 2.08 | 2.02 | 1.96 |
| 33 | 7.47 | 5.31 | 4.44 | 3.95 | 3.63 | 3.41 | 3.24 | 3.11 | 3.00 | 2.91 | 2.63 | 2.48 | 2.32 | 2.23 | 2.18 | 2.06 | 2.00 | 1.93 |
| 34 | 7.44 | 5.29 | 4.42 | 3.93 | 3.61 | 3.39 | 3.22 | 3.09 | 2.98 | 2.89 | 2.61 | 2.46 | 2.30 | 2.21 | 2.16 | 2.04 | 1.98 | 1.91 |
| 35 | 7.42 | 5.27 | 4.40 | 3.91 | 3.59 | 3.37 | 3.20 | 3.07 | 2.96 | 2.88 | 2.60 | 2.44 | 2.28 | 2.19 | 2.14 | 2.02 | 1.96 | 1.89 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.52 | 2.37 | 2.20 | 2.11 | 2.06 | 1.94 | 1.87 | 1.80 |
| 50 | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 | 2.70 | 2.42 | 2.27 | 2.10 | 2.01 | 1.95 | 1.82 | 1.76 | 1.68 |
| 75 | 6.99 | 4.90 | 4.05 | 3.58 | 3.27 | 3.05 | 2.89 | 2.76 | 2.65 | 2.57 | 2.29 | 2.13 | 1.96 | 1.87 | 1.81 | 1.67 | 1.60 | 1.52 |
| 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.22 | 2.07 | 1.89 | 1.80 | 1.74 | 1.60 | 1.52 | 1.43 |
| 200 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 | 2.41 | 2.13 | 1.97 | 1.79 | 1.69 | 1.63 | 1.48 | 1.39 | 1.28 |
| ∞ | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 | 2.32 | 2.04 | 1.88 | 1.70 | 1.59 | 1.52 | 1.36 | 1.25 | 1.00 |

# ANOVA Example

In our example, N=2,779 and $j$=5

So, $df_{NUM} = j - 1 = 4$ and $df_{DENOM} = N - j = 2{,}774$

With $\alpha$=0.05, the critical value of $F$ is 2.37

We can thus state a decision rule:

$H_0$: $\mu_{\text{Big City}} = \mu_{\text{Suburbs}} = \mu_{\text{Small City}} = \mu_{\text{Village}} = \mu_{\text{Farm}}$

$H_1$: Not all of the population group means are equal

Fail to reject $H_0$ if $F \leq 2.37$

Reject $H_0$ if $F > 2.37$

# ANOVA Example

Calculate the test statistic ... *F*

This is <u>not</u> something we do by hand for large samples

STATA Output for our example:

**ANOVA**

Think of self as liberal or conservative

| | Sum of Squares | df | Mean Square | F | Sig. |
|---|---|---|---|---|---|
| Between Groups | 35.229 | 4 | 8.807 | 4.807 | .001 |
| Within Groups | 5082.393 | 2774 | 1.832 | | |
| Total | 5117.622 | 2778 | | | |

$$F_{J-1,N-J} = \frac{\mathrm{SS_{BETWEEN}}/\mathrm{J}-1}{\mathrm{SS_{WITHIN}}/\mathrm{N}-\mathrm{J}}$$

# ANOVA Example

Compare the test statistic to the critical value

1. If the test statistic is larger than the critical value, then reject $H_0$ (with probability of $\alpha$ of doing so even though $H_0$ should not be rejected)
2. If the test statistic is less than or equal to the critical value, then do no reject $H_0$ (with probability of $\beta$ of doing so even though $H_0$ should be rejected)

We determined that our critical value of *F* is 2.37

We observed an *F* statistic of 4.807

$H_0$: $F \leq 2.37$ $H_1$: $F > 2.38$

<u>Conclusion</u>: Reject $H_0$

# ANOVA vs Difference in Means When J=2

We have now seen <u>two</u> techniques for comparing the mean of the continuous variable Y across two population groups

First, we can conduct a test of the hypothesis that $\mu_{j=1}=\mu_{j=2}$ using a *t* test

Second, we can conduct an ANOVA with *J*=2

We will get the same result. This is because

$$t_{df_{DENOM}} = \sqrt{F_{1,df_{DENOM}}}$$

# Worksheet

There are three delivery companies: A, B, & C

I had all three mail me 5 packages. Below are the number of days it took for me to get the packages

| | | | | | |
|---|---|---|---|---|---|
| A | 2 | 2 | 3 | 4 | 6 |
| B | 1 | 2 | 2 | 5 | 5 |
| C | 2 | 2 | 3 | 3 | 4 |

**Hint/Help**:

$SS_{Between} = 0.9333$

$SS_{Within} = 28$

Test the hypothesis that the mean number of days that each company takes to deliver packages is equal; use $\alpha=0.05$ *(For this example, relax/ignore the basic assumptions that must be met in order to perform ANOVA)*

# Worksheet

There are three delivery companies: A, B, & C

I had all three mail me 3 packages. Below are the number of days it took for me to get the packages

| | | | |
|---|---|---|---|
| A | 1 | 1 | 2 |
| B | 2 | 2 | 3 |
| C | 3 | 3 | 4 |

**Hint/Help**:

$SS_{Between} = 6$

$SS_{Within} = 2$

Test the hypothesis that the mean number of days that each company takes to deliver packages is equal; use $\alpha$=0.05 *(For this example, relax/ignore the basic assumptions that must be met in order to perform ANOVA)*

# Want More?

David Lane's Book

- http://onlinestatbook.com/2/analysis_of_variance/ANOVA.html

Chapters 13 and 14 of Richard Lowry's Book

- http://vassarstats.net/textbook/

Gerard Dallal's Book

- http://www.jerrydallal.com/LHSP/anova1.htm
- http://www.jerrydallal.com/LHSP/aov1out.htm