SOC 3811/5811: BASIC SOCIAL STATISTICS

Hypothesis Testing

Hypothesis Tests

We will consider four different sorts of hypothesis tests, all of which follow the same logic

1. **Hypothesis Tests for <u>Proportions</u>**

   Use $\hat{p}$ to infer p, the population proportion

2. **Hypothesis Tests for <u>Means</u>**

   Use $\bar{Y}$ to infer μ_Y, the population mean of Y

3. **Hypothesis Tests for <u>Differences in Proportions</u>**

   Use $\hat{p}_1 - \hat{p}_2$ to infer the difference between two population proportions, p_1 and p_2

4. **Hypothesis Tests for <u>Differences in Means</u>**

   Use $\bar{Y}_1 - \bar{Y}_2$ to infer the difference between two population means, μ_{Y1} and μ_{Y2}

How to Test a Hypothesis

1. State the null (H_0) and alternative (H_1) hypotheses
2. Check that the sample data conform to basic assumptions; if they do not, then do not go any further
3. Choose an α probability level … that is, a probability associated with incorrectly rejecting the null hypothesis
4. Determine the "critical value" … that is, how large the test statistic must be in order to reject the null hypothesis at the given α level
5. Calculate the test statistic (t or Z)
6. Compare the test statistic to the critical value
   - If the test statistic is larger than the critical value, then reject H_0 (with probability of α of doing so even though H_0 should not actually be rejected)
   - If the test statistic is less than or equal to the critical value, then do not reject H_0 (with probability of β of doing so even though H_0 should be rejected)

How to Test a Hypothesis

TEST STATISTICS

For hypotheses about a *proportion*

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{N}}}$$

For hypotheses about a *mean*

$$t = \frac{\bar{Y} - \mu_Y}{s_Y/\sqrt{n}}$$

For hypotheses about *differences in proportions*

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\hat{p}(1 - \hat{p})}{n_1} + \frac{\hat{p}(1 - \hat{p})}{n_2}}} \quad \text{where } \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

For hypotheses about *differences in means*

$$t = \frac{\bar{Y}_M - \bar{Y}_F - 0}{\sqrt{\frac{s_M^2}{n_M} + \frac{s_F^2}{n_F}}}$$

Worksheet

Were Minnesota kids more likely to be in families that received food stamps in 2010 than in 2007?

Data from 2007 and 2010 American Community Surveys:

2007: 7.1% of 13,511 sampled MN kids in such families

2010: 11.7% of 12,832 sampled MN kids in such families

1. Construct a 99% confidence interval for the difference in proportions between these two years
2. Test the hypothesis that there was no difference between the two years in the rate of food stamp receipt. Use an α of 0.01.

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$$\hat{p}_1 - \hat{p}_2 \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

Standard Normal Probabilities

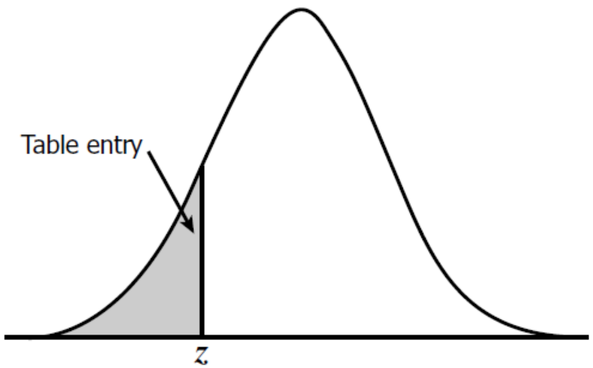

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
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−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
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−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681

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$$0.71 - 0.117 \pm 2.575\sqrt{\frac{0.071(0.929)}{13511} + \frac{0.117(0.883)}{12{,}832}} = -0.046 \pm 0.009; -0.055\ to\ -0.037$$

Worksheet

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2. Test the hypothesis that there was no difference between the two years in the rate of food stamp receipt. Use an α of 0.01.

1. State the null (H_0) and alternative (H_1) hypotheses

Because we are asking whether or not the difference in proportions is different from a particular value (zero), this is a two-sided hypothesis test

H_0: $p_{2007}=p_{2012}$ (or, $p_{2012}-p_{2007}=0$)

H_1: $p_{2007}\neq p_{2012}$ (or, $p_{2012}-p_{2007}\neq 0$)

2. Check that the sample data conform to basic assumptions; if they do not, then do not go any further

The two samples must be independent and representative of the two populations from which they were drawn (True)

As with confidence intervals, $n_1 \hat{p}_1$, $n_1(1- \hat{p}_1)$, $n_2 \hat{p}_2$, and $n_2(1- \hat{p}_2)$ must all exceed 5 (and preferably 10) (True)

3. Choose an α probability level … that is, a probability associated with incorrectly rejecting the null hypothesis

α=0.01 in this example

4. Determine the "critical value" … that is, how large the test statistic must be in order to reject the null hypothesis at the given α level

Because we are dealing with sample proportions, our test statistic will be a Z-score

What critical value Z* corresponds to having 1%/2=0.5% of the area under the normal curve above +Z* and 0.5% of the area under the normal curve below –Z*?

Standard Normal Probabilities

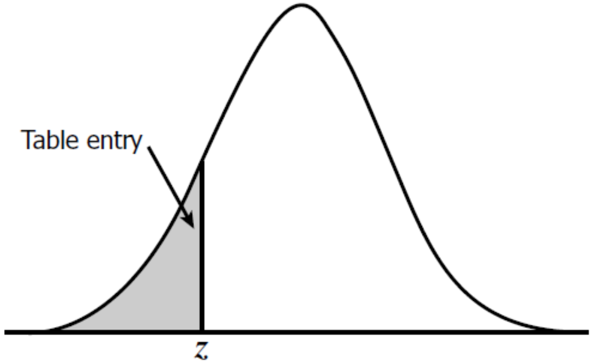

Table entry for *z* is the area under the standard normal curve to the left of *z*.

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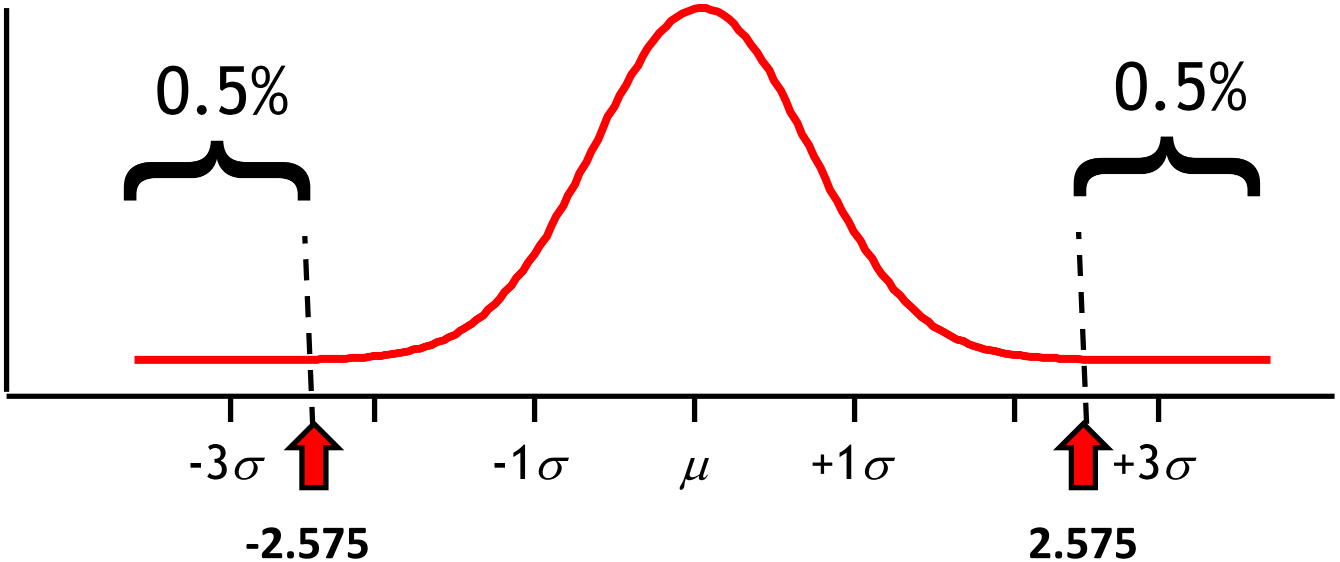
With α=0.01 and a two-sided test …



… so our critical value Z* is 2.575

H_0: $p_{2007}=p_{2012}$ … fail to reject H_0 if $|Z| \leq 2.575$

H_1: $p_{2007} \neq p_{2012}$ … reject H_0 if $|Z| > 2.575$

5. Calculate the test statistic

All test statistics for hypothesis testing…

…subtract the population value that is assumed to be true under the null from the observed (sample) value

…divide that figure by the standard deviation of the sampling distribution of the statistic in question

For differences in proportions:

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}}} \quad \text{where} \quad \hat{p} = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$$

In our example: $\hat{p}_{2007} = 0.071$ $n_{2007} = 13{,}511$

$\hat{p}_{2012} = 0.117$ $n_{2012} = 12{,}832$

$$\hat{p} = \frac{n_{2007}\hat{p}_{2007} + n_{2012}\hat{p}_{2012}}{n_{2007} + n_{2012}} =$$

$$Z = \frac{\hat{p}_{2007} - \hat{p}_{2012} - 0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_{2007}} + \frac{\hat{p}(1-\hat{p})}{n_{2012}}}} =$$

In our example: $\hat{p}_{2007} = 0.071$ $n_{2007} = 13{,}511$

$\hat{p}_{2012} = 0.117$ $n_{2012} = 12{,}832$

$$\hat{p} = \frac{n_{2007}\hat{p}_{2007} + n_{2012}\hat{p}_{2012}}{n_{2007} + n_{2012}} = \frac{13{,}511(0.071) + 12{,}832(0.117)}{13{,}511 + 12{,}832} = 0.093$$

$$Z = \frac{\hat{p}_{2007} - \hat{p}_{2012} - 0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_{2007}} + \frac{\hat{p}(1-\hat{p})}{n_{2012}}}} = \frac{0.071 - 0.117 - 0}{\sqrt{\frac{0.093(1-0.093)}{13{,}511} + \frac{0.093(1-0.093)}{12{,}832}}} = 12.85$$

6. Compare the test statistic to the critical value

- If the test statistic is larger than the critical value, then reject H_0 (with probability of α of doing so even though H_0 should not actually be rejected)
- If the test statistic is less than or equal to the critical value, then do not reject H_0 (with probability of β of doing so even though H_0 should be rejected)

In our example, our critical value Z^* is 2.575

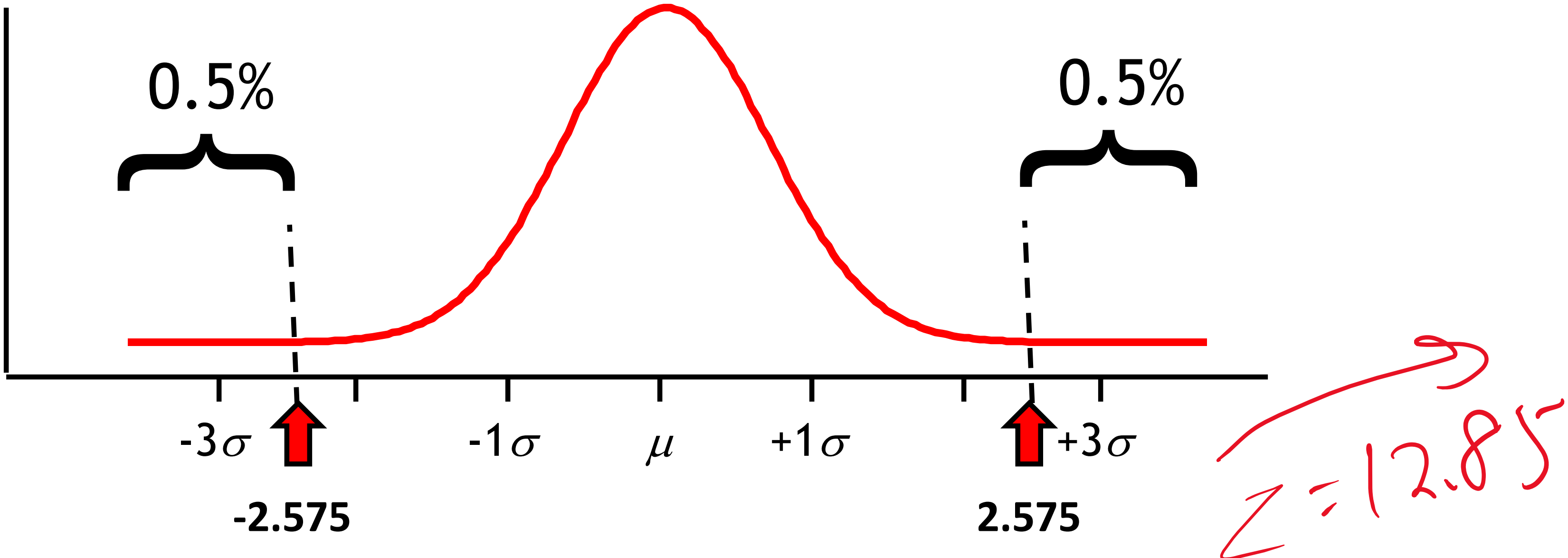
We obtained a test statistic of 12.85

H_0: $p_{2007}=p_{2012}$ … fail to reject H_0 if $|Z| \leq 2.575$

H_1: $p_{2007} \neq p_{2012}$ … reject H_0 if $|Z| > 2.575$

Conclusion? Reject H_0

With α=0.01 and a two-sided test …



… so our critical value Z* is 2.575

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Worksheet

Were Minnesota kids more likely to be in families that received food stamps in 2010 than in 2007?

1. Construct a 99% confidence interval for the difference in proportions between these two years

   -0.55 to -0.37

2. Test the hypothesis that there was no difference between the two years in the rate of food stamp receipt. Use an α of 0.01. **Reject the null hypothesis that there was no difference**

Standard Normal Probabilities

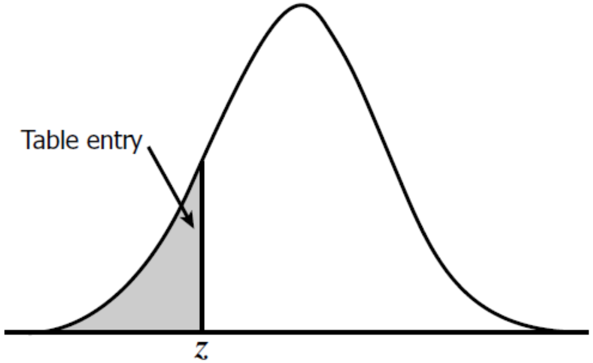

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TABLE D

t distribution critical values

	Upper-tail probability *p*											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300