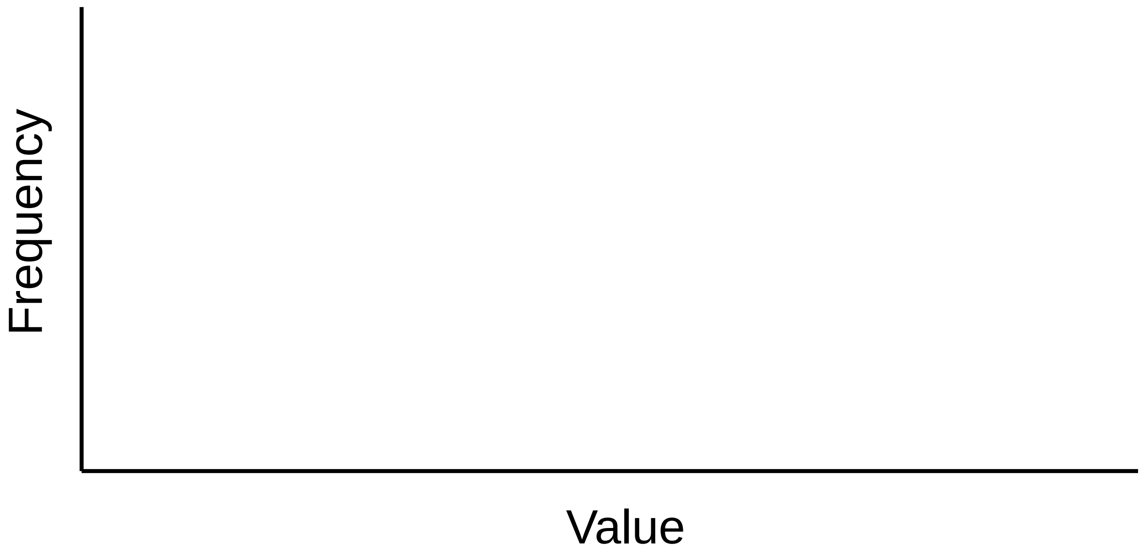
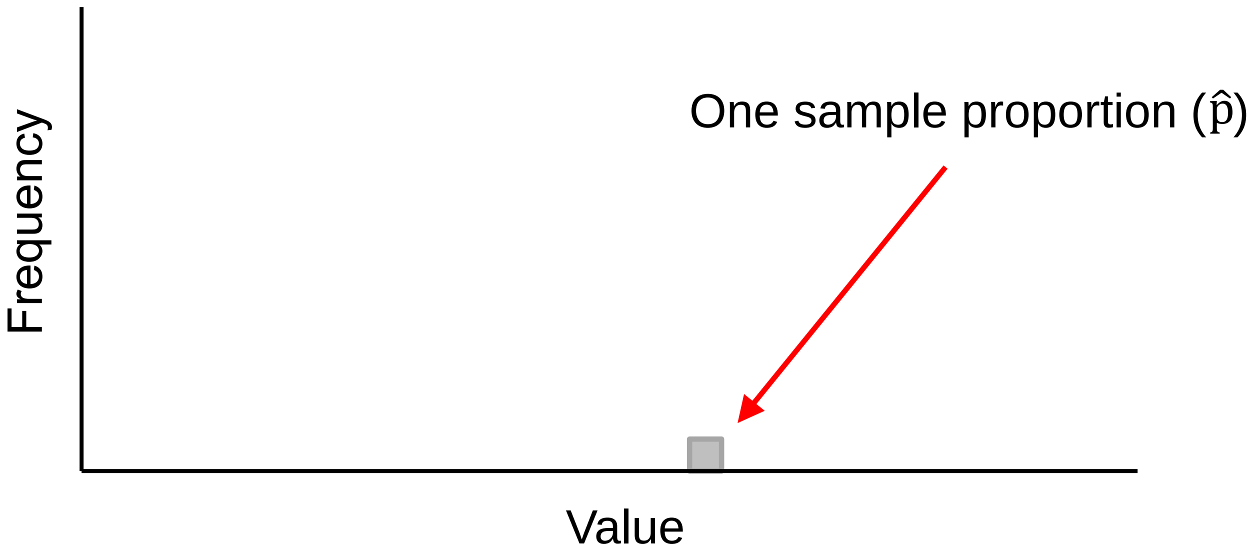
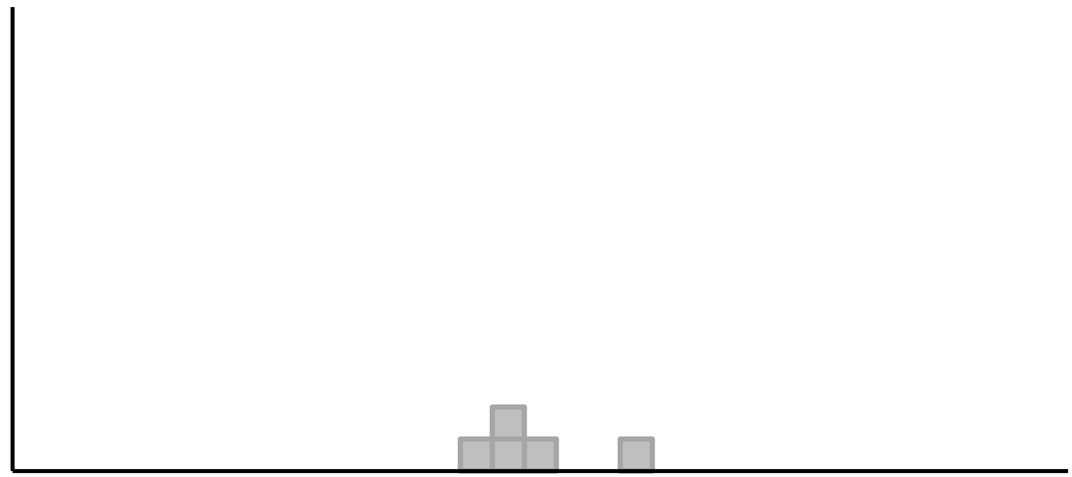
SOC 3811/5811: BASIC SOCIAL STATISTICS

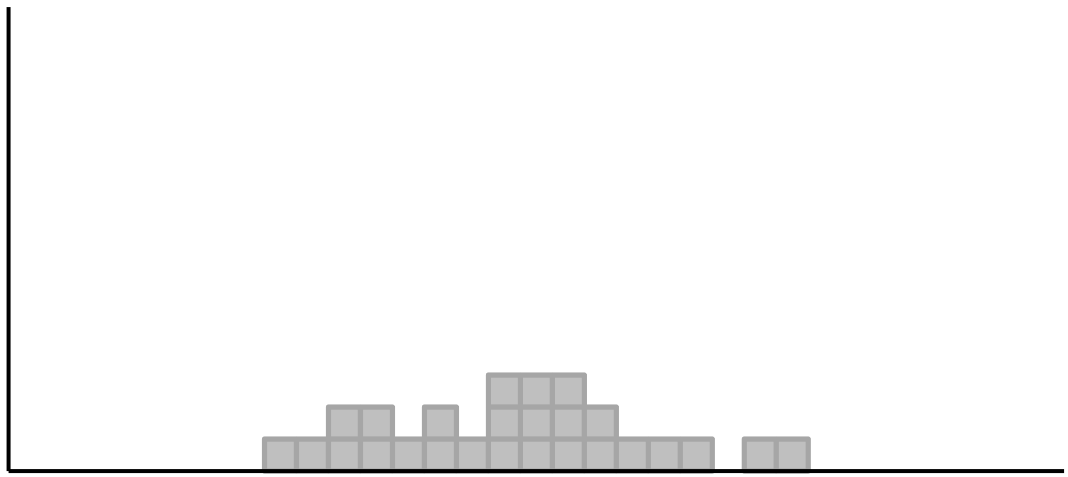
Hypothesis Testing

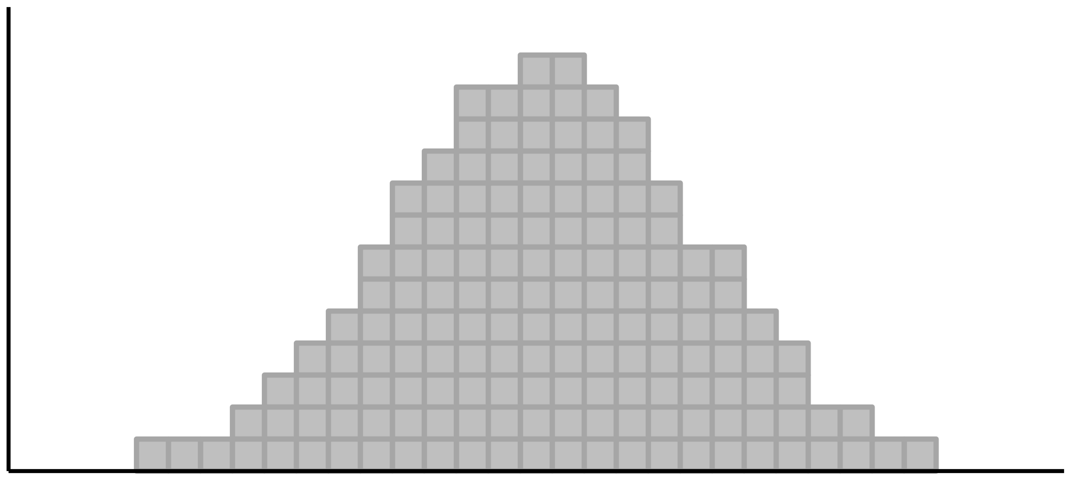

Frequency
Value

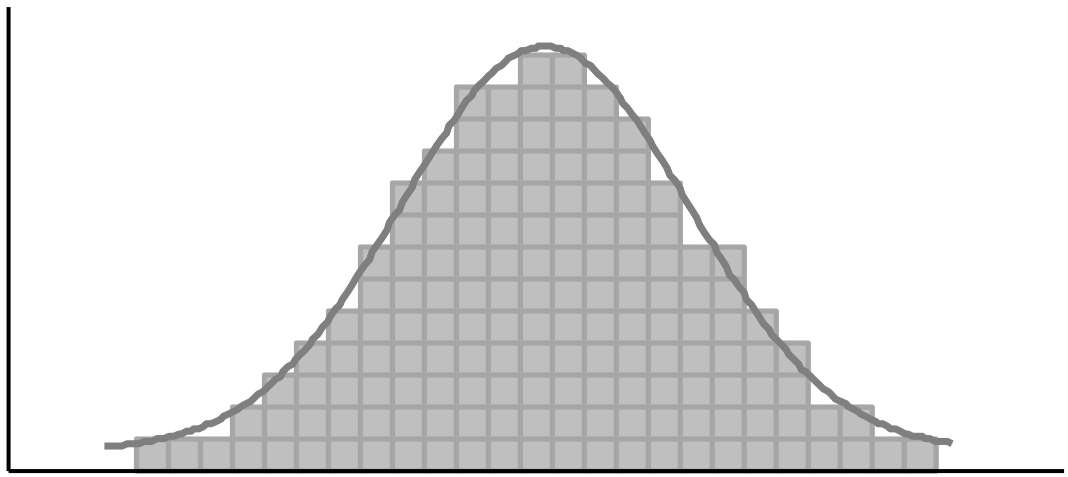


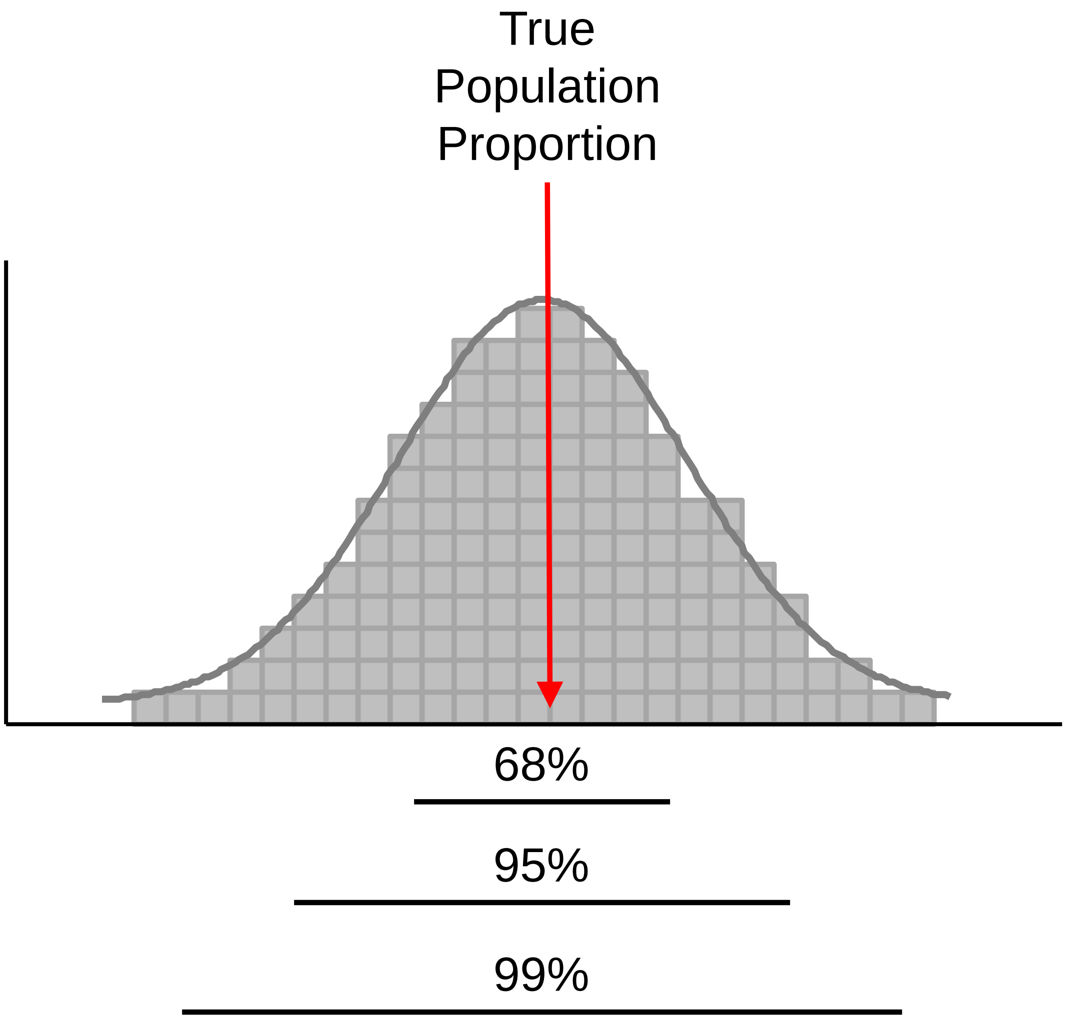
One sample proportion ($\hat{p}$)
Frequency
Value



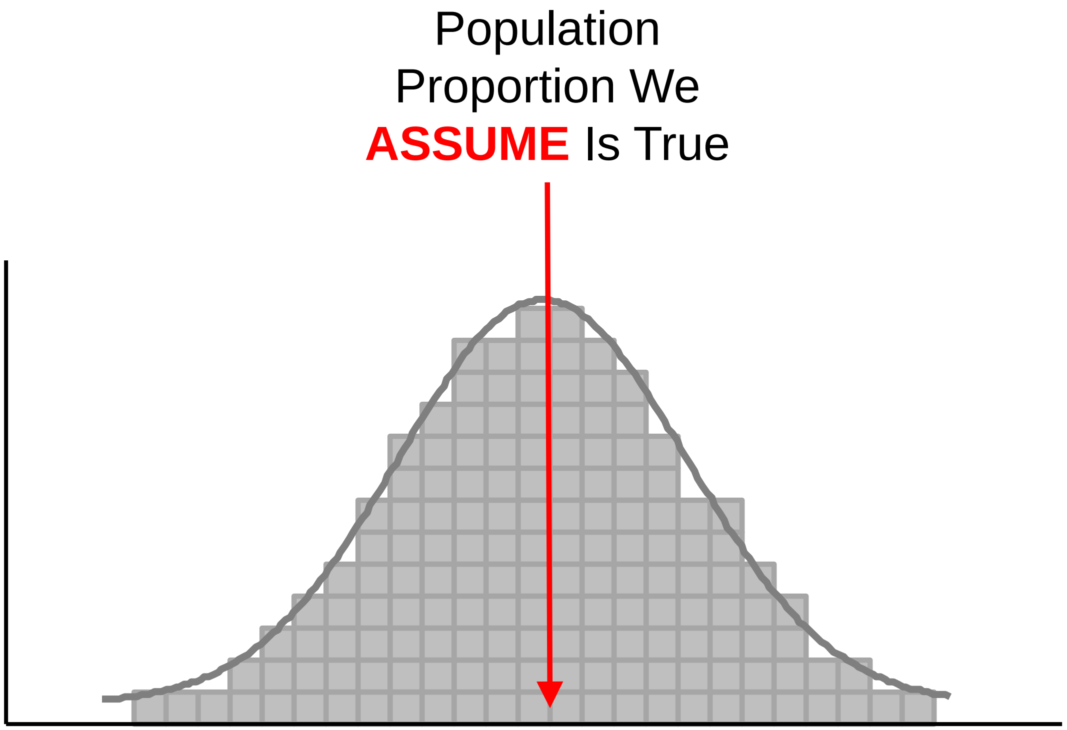




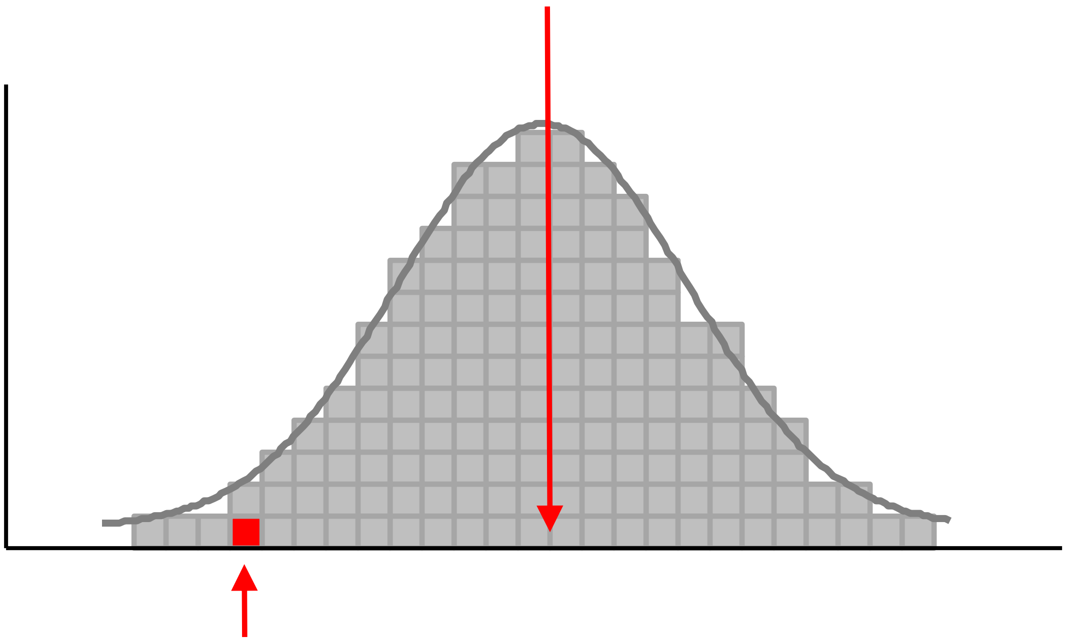



True
Population
Proportion
68%
95%
99%

Hypothesis Testing


Population
Proportion We
ASSUME Is True

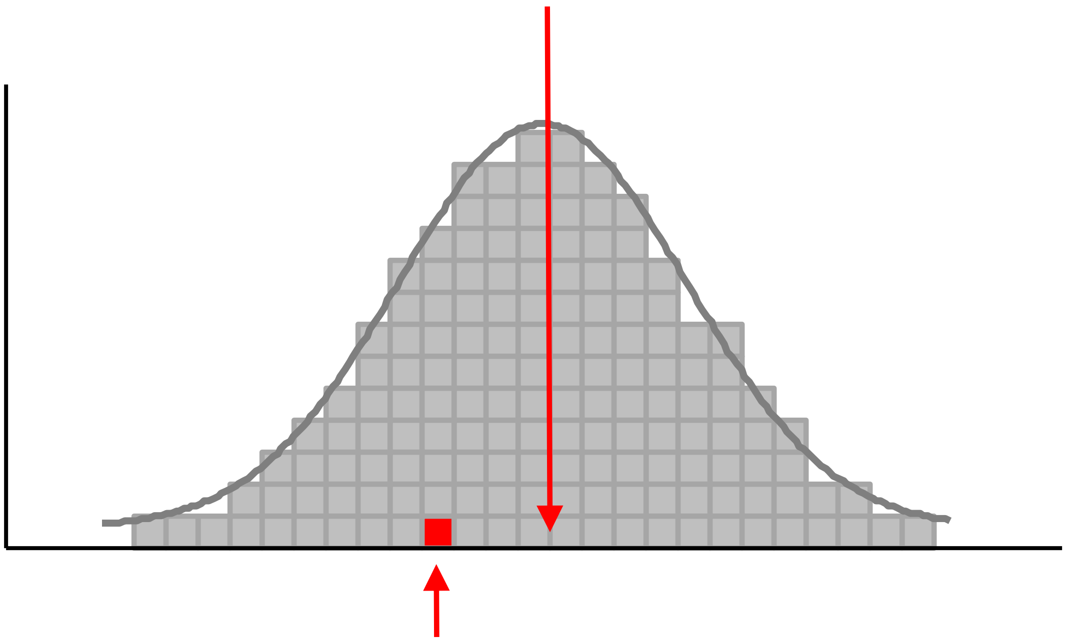
Population
Proportion We
ASSUME Is True



What is the probability of getting a sample proportion THIS different from the population proportion that we assume to be true?

If it is LOW, we reject our assumption about the value of the population proportion

Population
Proportion We
ASSUME Is True



What is the probability of getting a sample proportion THIS different from the population proportion that we assume to be true?

If it is HIGH, we do not reject our assumption about the value of the population proportion

Hypothesis Testing

Hypothesis tests involve two competing (mutually exclusive and exhaustive) hypotheses; what these hypotheses are depends on the question at hand

Examples:

Hyp. 1: More than 95% of all U.S. drivers use their seat belts

Hyp. 2: 95% or fewer of all U.S. drivers use their seat belts

Hyp. 1: There are 12.1 penguins per square mile

Hyp. 2: The number of penguins per square mile is not 12.1

Hypothesis Testing

These two hypotheses are typically called…

…the "null" hypothesis (H_0) and

…the "alternative" hypothesis (H_1 or H_A … H_1 in this class)

The **null hypothesis** often says that nothing is happening or that something is not true

The **alternative hypothesis** usually says that something is happening or that something is true

Hypotheses

More than 50% of Minnesota voters support a $15 per hour statewide minimum wage

H_0: H_a:

The typical U of MN students works fewer than 20 hours per week at their paid jobs

H_0: H_a:

5% of American taxpayers were audited by the IRS last year

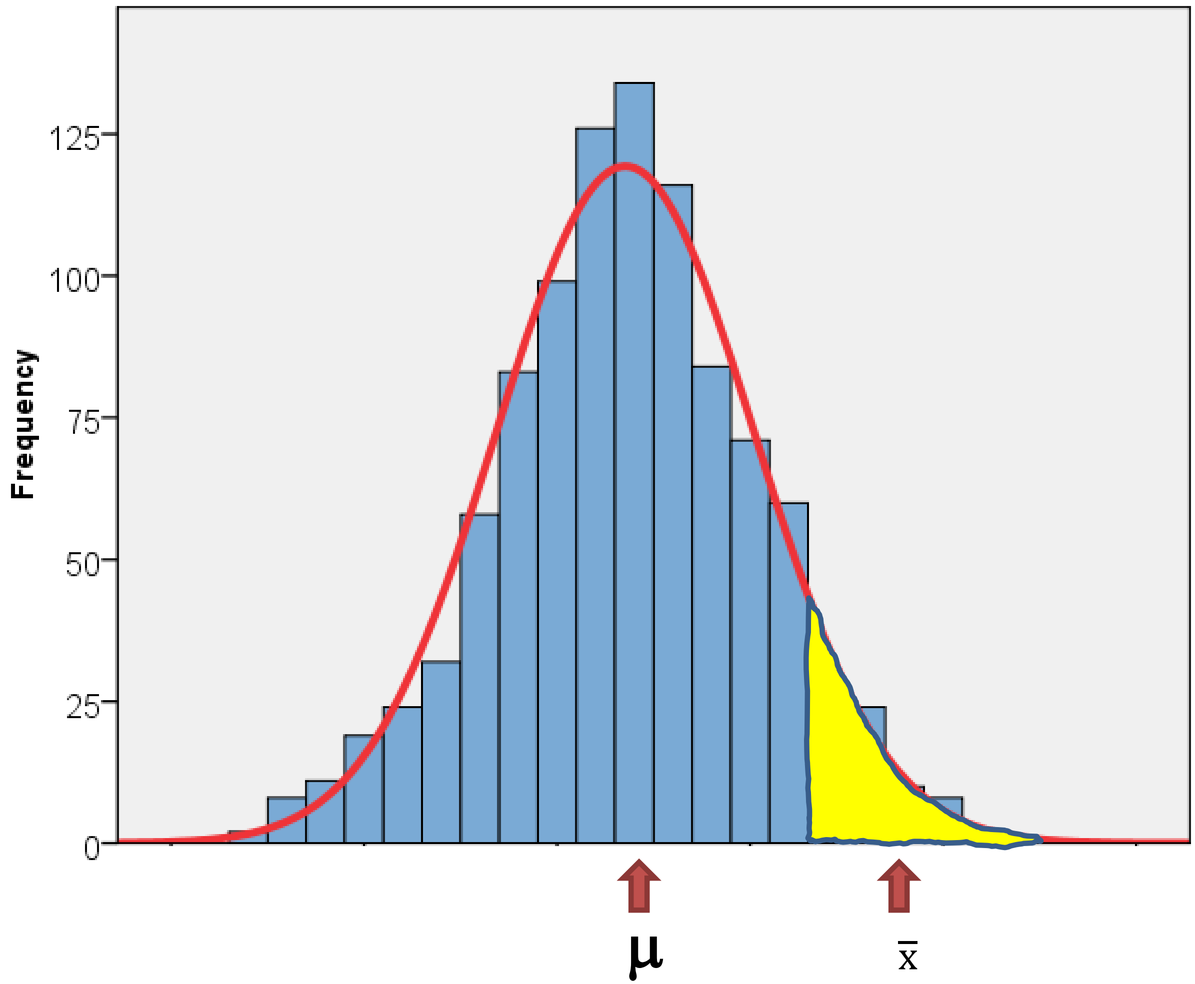
H_0: H_a:

The average weight of an Oreo® is 11.3 grams

H_0: H_a:

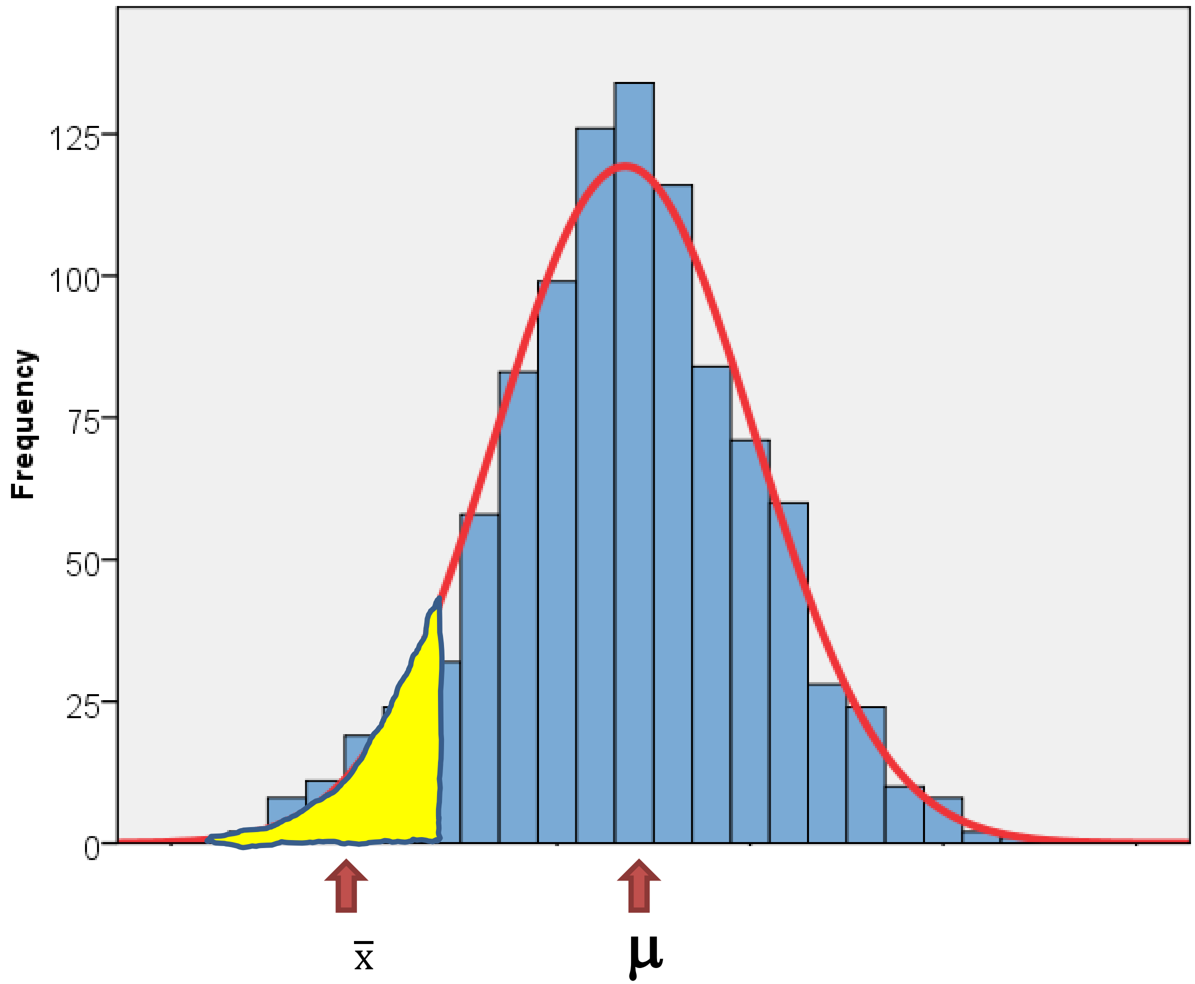
One-sided (upper tail) hypothesis test

Assuming that μ is true, what is the probability of observing one sample mean that far **above** μ?



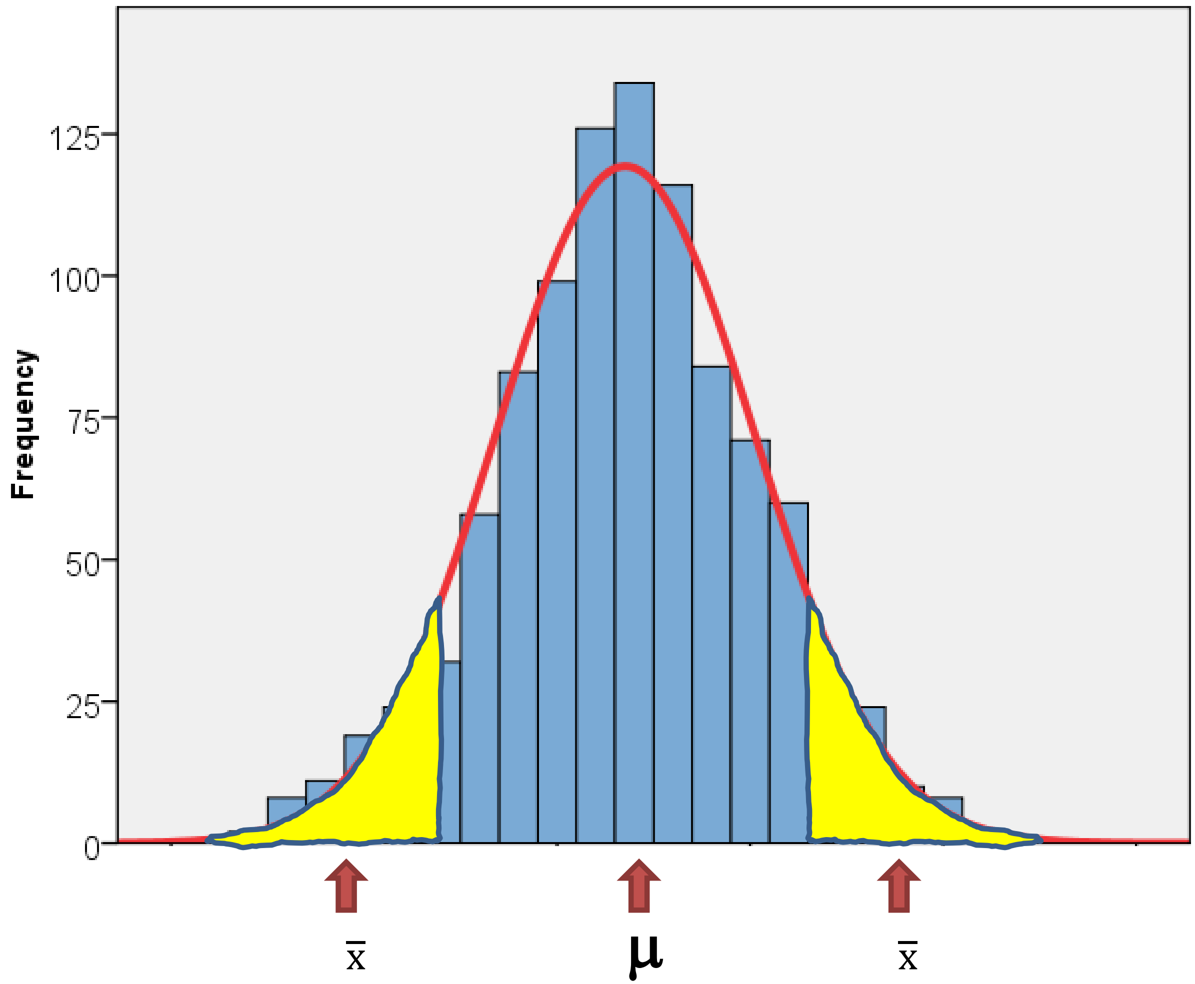
One-sided (lower tail) hypothesis test

Assuming that μ is true, what is the probability of observing one sample mean that far **below** μ?



Two-sided hypothesis test

Assuming that μ is true, what is the probability of observing one sample mean that far **above or below** μ?



Hypothesis Testing

1. State the null (H_0) and alternative (H_1) hypotheses
2. Check that the sample data conform to basic assumptions; if they do not, then do not go any further
3. Choose an α probability level … that is, a probability associated with incorrectly rejecting the null hypothesis
4. Determine the "critical value" … that is, how large the test statistic must be in order to reject the null hypothesis at the given α level
5. Calculate the test statistic
6. Compare the test statistic to the critical value
   - If the test statistic is as large as the critical value, then reject H_0 (with probability of α of doing so even though H_0 should not actually be rejected)
   - If the test statistic is less than or equal to the critical value, then do not reject H_0 (with probability of β of doing so even though H_0 should be rejected)

Hypothesis Testing

We always operate under the assumption that the null hypothesis is true in the population

Given that the null hypothesis is assumed to be correct …

…if the probability of observing the sample data (that is, the observed sample proportion, the observed sample mean, etc.) is very low then we reject the null

“Rejecting” and “failing to reject” the null hypothesis are the only possible outcomes of a hypothesis test

We <u>always</u> operate under the assumption that the null hypothesis is true in the population

Example 1

A college set a goal of having more than 90% of its alumni get jobs immediately after graduation

To see whether they are on target for meeting that goal, they randomly sampled 1,000 alumni

They found that 912—or 91.2%—of the 1,000 alumni they sampled got jobs immediately after graduation

Is this evidence sufficient to confidently conclude that more than 90% of <u>all</u> alumni get jobs immediately after graduation?

Example 1

1. State the null (H_0) and alternative (H_1) hypotheses

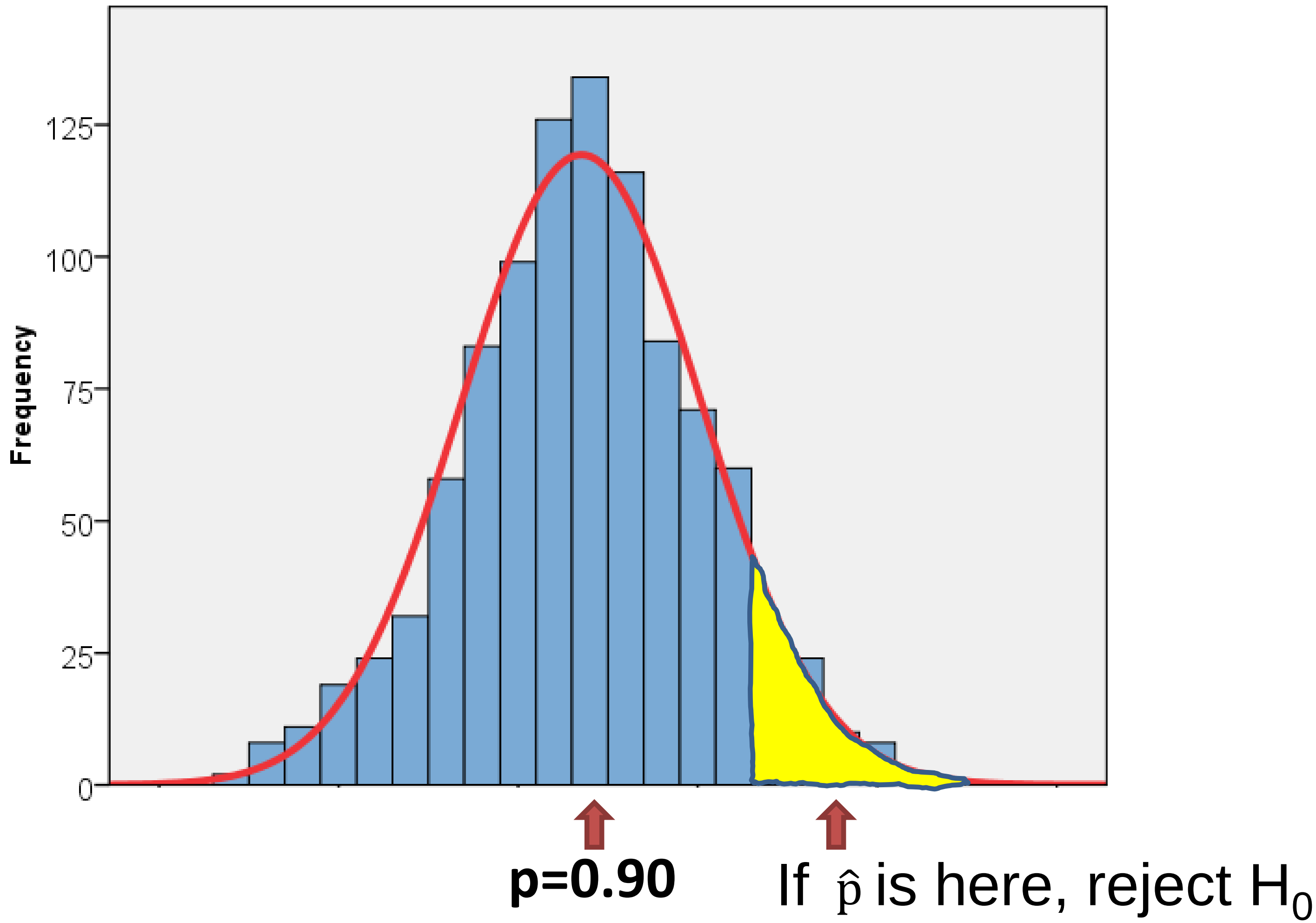
Because we are asking whether the proportion exceeds a particular value (instead of just being different from it), this is a one-sided hypothesis test

H_0: $p \leq 0.90$

H_1: $p > 0.90$

One-sided (upper tail) hypothesis test

Assuming that p=0.90 or less, what is the probability of observing one sample proportion that far **above** p?



Example 1

2. Check that the sample data conform to basic assumptions; if they do not, then do not go any further

The sample must have been randomly selected from the population … (True)

np_0 and $n(1-p_0)$ must both be at least 10 (where p_0 is the proportion we assume to be correct based on the null hypothesis)

$np_0=1{,}000\times0.90=900$ and $n(1-p_0) = 1{,}000\times0.10=100$

(True)

Example 1

3. Choose an α probability level … that is, a probability associated with incorrectly rejecting the null hypothesis

The precise α level we select is arbitrary, and varies by discipline (and sometimes by journal or even by author)

0.05 is most common; 0.01 is common, too

Let's go with α=0.01 in this example

Example 1

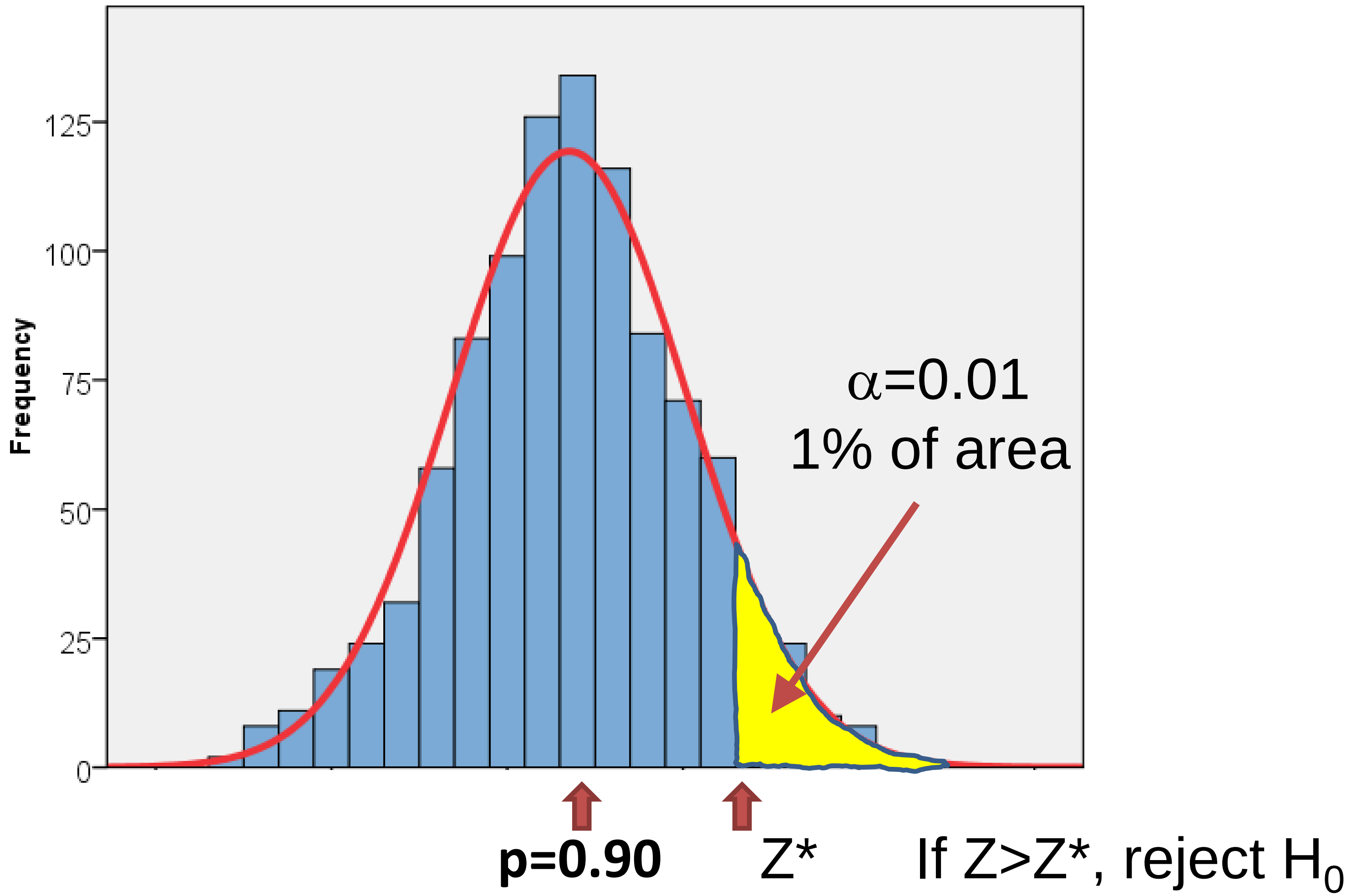
4. Determine the "critical value" … that is, how large the test statistic must be in order to reject the null hypothesis at the given α level

How large must the critical value Z* be in order to have the area under the standard normal curve that exceeds Z* be less than α?

It can then be helpful to re-write the hypotheses in terms of the Z-score(s)

One-sided (upper tail) hypothesis test

Assuming that p=0.90 or less, what is the probability of observing one sample proportion that far **above** p?



Standard Normal Probabilities

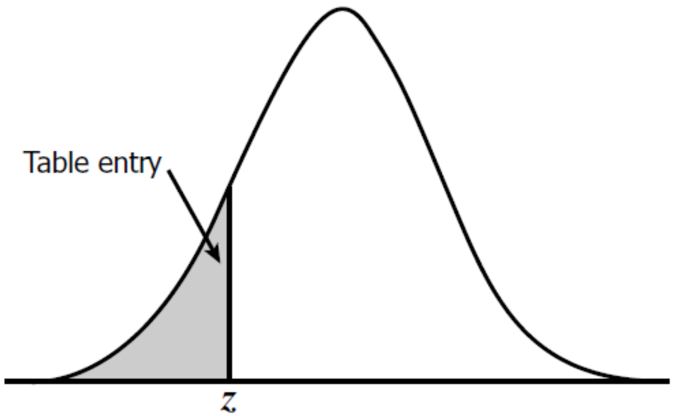
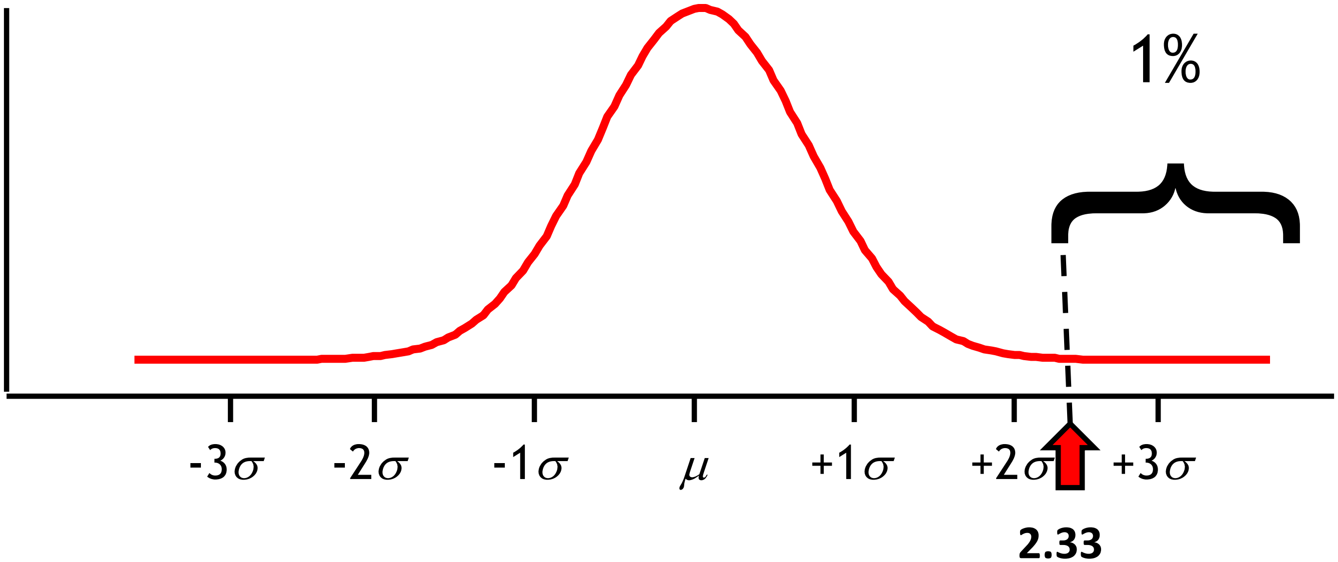


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233

Example 1

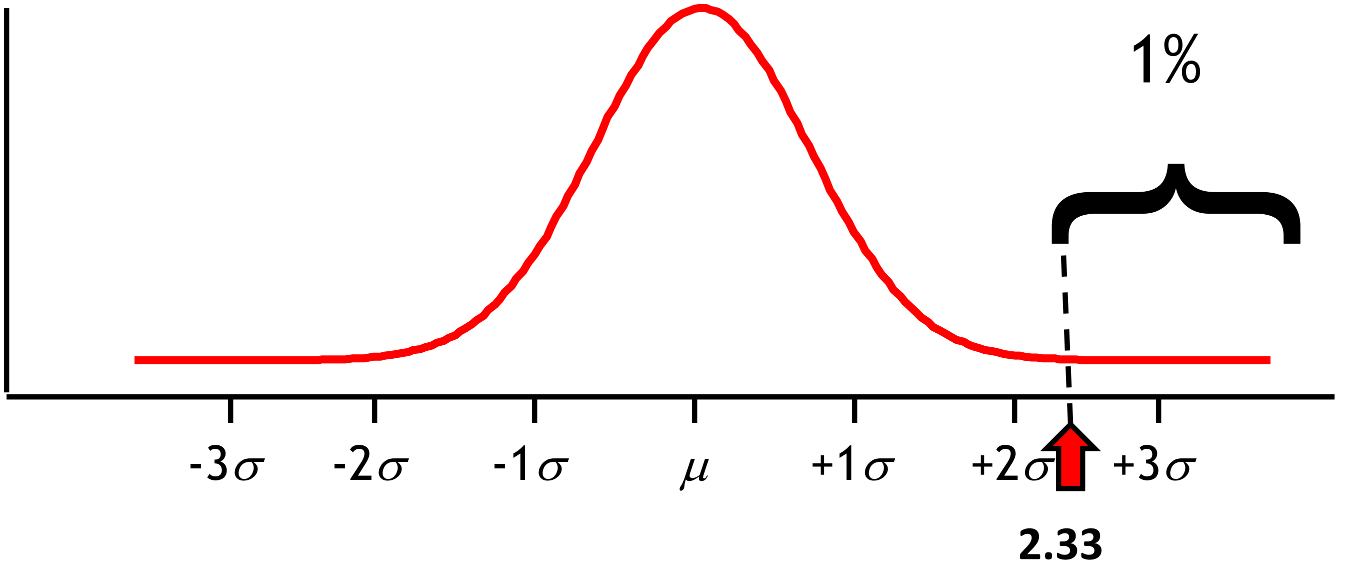
With α=0.01 and a one-sided test …



… a Z-score that is 2.33 has 1% of the area under the curve beyond it

Example 1

We are using α=0.01 and a one-sided test …



… so our critical value Z* is +2.33

H_0: p ≤ 0.90 … fail to reject H_0 if test statistic Z ≤ 2.33

H_1: p > 0.90 … reject H_0 if test statistic Z > 2.33

Example 1

5. Calculate the test statistic

All test statistics for hypothesis testing…

…subtract the population value that is assumed to be true under the null from the observed (sample) value

…divide that figure by the standard deviation of the sampling distribution of the statistic in question

For proportions:

$$Z = \frac{\text{sample estimate} - \text{null value}}{\text{null standard deviation}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

Example 1

In our example:

H_0: p ≤ 0.90

H_1: p > 0.90

$$\mathrm{n} = 1{,}000$$

$$\hat{\mathrm{p}} = \frac{912}{1{,}000} = 0.912$$

$$\mathrm{Z} = \frac{\hat{\mathrm{p}} - \mathrm{p}_0}{\sqrt{\frac{\mathrm{p}_0(1-\mathrm{p}_0)}{\mathrm{N}}}} =$$

Example 1

In our example:

H_0: p ≤ 0.90

H_1: p > 0.90

$$n = 1{,}000$$

$$\hat{p} = \frac{912}{1{,}000} = 0.912$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{N}}} = \frac{0.912 - 0.900}{\sqrt{\frac{0.900(1-0.900)}{1{,}000}}} = 1.26$$

Example 1

6. Compare the test statistic to the critical value

- If the test statistic is larger than the critical value, then reject H_0 (with probability of α of doing so even though H_0 should not actually be rejected)
- If the test statistic is less than or equal to the critical value, then do no reject H_0 (with probability of β of doing so even though H_0 should be rejected)

In our example, our critical value Z* is +2.33

We obtained a test statistic of 1.26

H_0: $p \leq 0.90$ … fail to reject H_0 if test statistic $Z \leq 2.33$

H_1: $p > 0.90$ … reject H_0 if test statistic $Z > 2.33$

Conclusion? Fail to Reject H_0

Errors in Hypothesis Testing

When we test a hypothesis, we can come to one of two conclusions

Reject the null hypothesis (H_0)

Fail to reject the null hypothesis (H_0)

Regardless of what we decide, we risk making an error

If we reject H_0, it could be that H_0 is actually true … such that we should not have rejected it

If we fail to reject H_0, it could be that H_0 is actually false … such we should have rejected it

Errors in Hypothesis Testing

THE TRUTH IS…

WE CONCLUDE…

	H_0 is True	H_0 is False
Fail to Reject H_0	✓	✗ Type II Error *P*(Type II Error)=β
Reject H_0	✗ Type I Error *P*(Type I Error)=α	✓

Errors in Hypothesis Testing

What are the consequences of both types of errors?

The new drug we developed is effective

H_0: Drug doesn't work H_a: Drug works

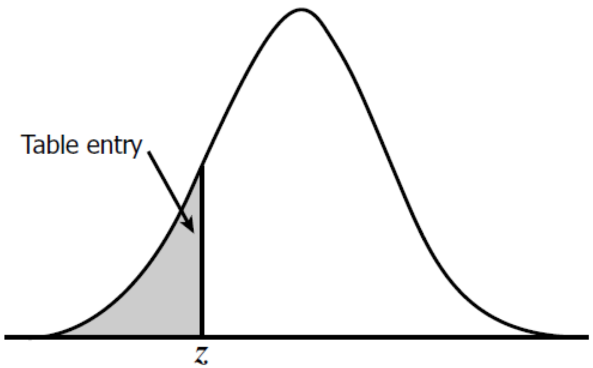
The average weight of an Oreo® is 11.3 grams

H_0: $\mu=11.3$ H_a: $\mu \neq 11.3$

The 4th graders are reading at or above the grade level expectation of 150 words per minute

H_0: $\mu \leq 150$ H_a: $\mu > 150$

Standard Normal Probabilities



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−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681