SOC 3811/5811:
BASIC SOCIAL STATISTICS

Continuous Random Variables

Binomial random variable Y equals the number of baseball games won out of n=18 independent binomial events/games when the probability of winning any one game is taken to be p=0.5

The probability that Y = k (where k ranges from 0 to n) is:

$$P(Y=k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

Thus, the probability of winning k=0 baseball games out of n=18 games is:

$$P(Y = 0) = \frac{18!}{0!(18-0)!} 0.5^0 (1-0.5)^{18-0} = (1)(1)(0.5^{18})$$

$$= \text{about } 0.000004 = \text{about 1 in 250,000}$$

Continuous Random Variables

As before, these are theoretical distributions ... distributions of Y if we were to sample from the population an infinite number of times

As with all distributions, we can describe distributions with respect to their central tendency and amount of variability

Normal Random Variables

Just as a binomial random variables is a special (and very common) kind of discrete random variable, a **normal random variables** is a special (and very common) kind of continuous random variable
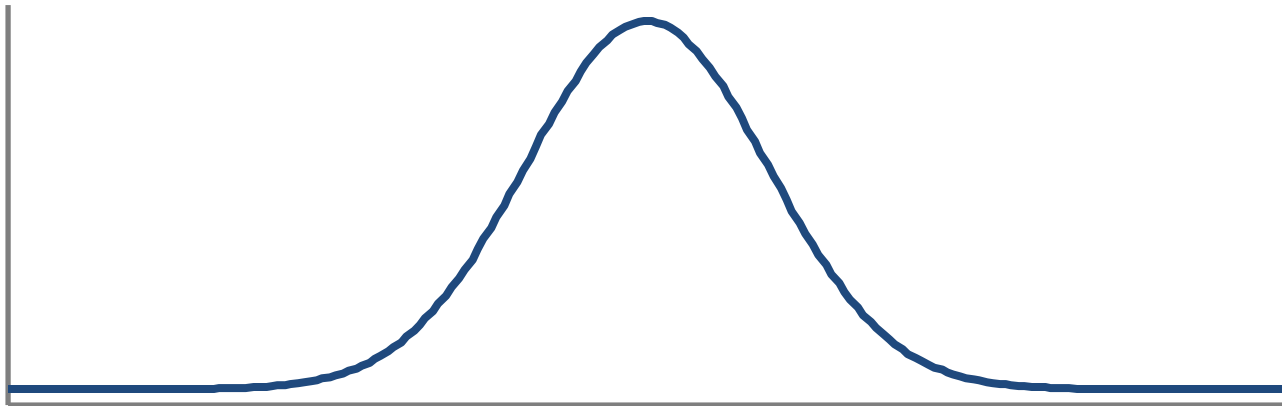
Normal Random Variables

Any normal random variable Y is symmetric and can be characterized by its mean μ_Y and standard deviation σ_Y

Remember Z scores? $Z = \dfrac{(Y - \bar{Y})}{s_Y}$

For *any* normally distributed random variable:
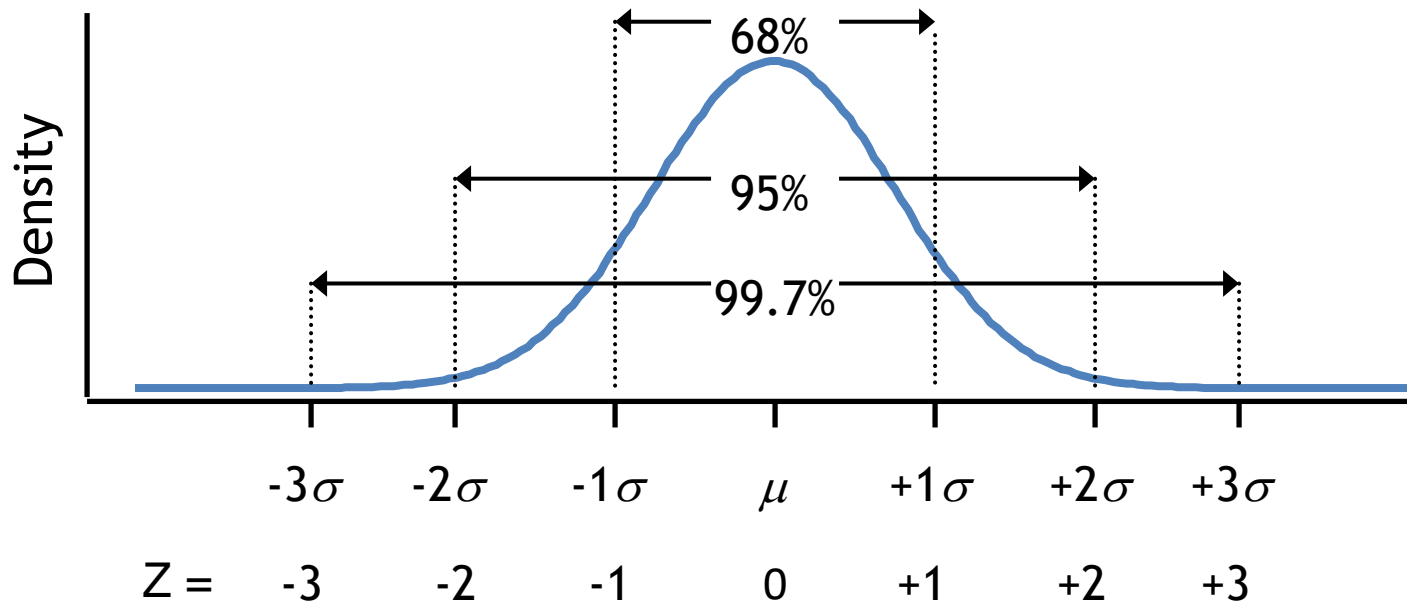
~68% of cases fall within the range -1Z and +1Z
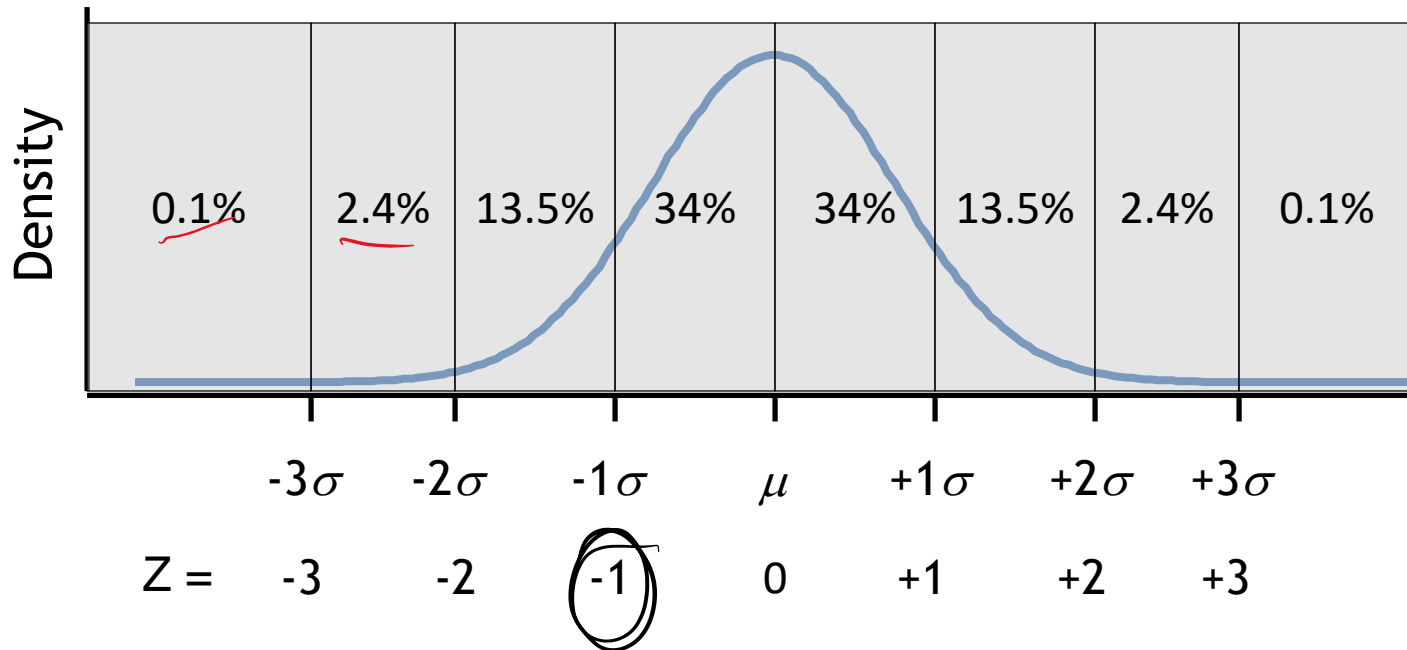
~95% of cases fall within the range -2Z and +2Z

~99.7% of cases fall within the range -3Z and +3Z

100% of cases fall within the range $-\infty Z$ and $+ \infty Z$

Normal Random Variables

Normal Random Variables

PRACTICE USING Z TABLE

What is the area under the normal curve…

…to the left of Z=-1 … or P(Z<-1)

…to the right of Z=-1 … or P(Z>-1)

…to the left of Z=.32 … or P(Z<.32)

…between Z=-.1 and Z=.6 … or P(.1<Z<.6)

PRACTICE USING Z TABLE

What is the area under the normal curve…

…to the left of Z=-1 … or P(Z<-1)   = 15.87%

…to the right of Z=-1 … or P(Z>-1)   = 84.13%

…to the left of Z=.32 … or P(Z<.32)   = 62.55%

…between Z=-.1 and Z=.6 … or P(-.1<Z<.6)

=0.7257 − 0.4602 = 0.2655 = 26.55%

PUTTING Z TABLE TO USE

Grades in my class are a normal random variable with mean μ=3.2 and standard deviation σ=.2. What percentage of students earn a grade in my class...

...less than 2.5?

...more than 3.0?

...between 3.0 and 3.5?
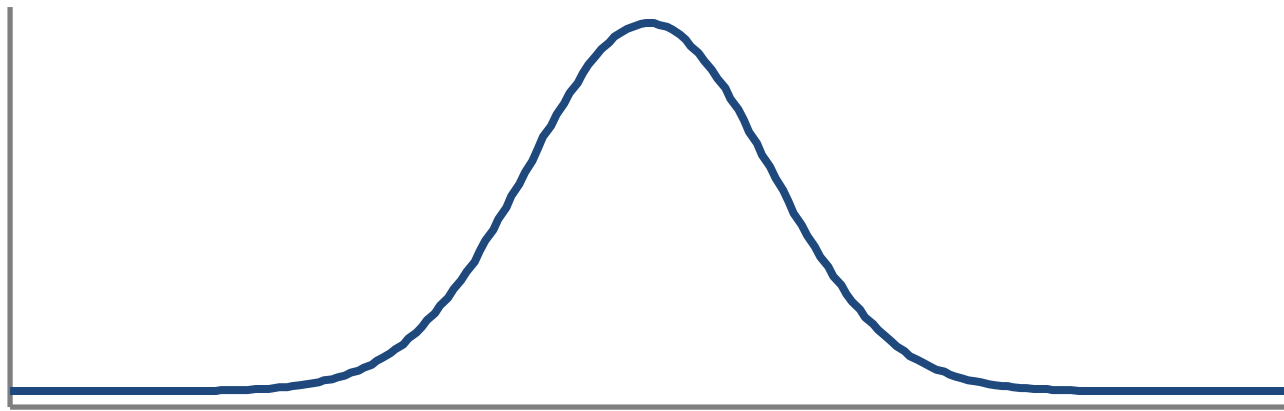
PUTTING Z TABLE TO USE

Grades in my class are a normal random variable with mean μ=3.2 and standard deviation σ=.2. What percentage of students earn a grade in my class…

…less than 2.5?   = P(Z<-3.5) = <.02%

…more than 3.0?   = P(Z>-1) = 84.13%

…between 3.0 and 3.5?   = P(-1<Z<1.5)

= 0.9332 − 0.1587 = 0.7745 = 77.45%

Combinations of Random Variables

It is often useful to combine (e.g., add or subtract) random variables

Example: The weight of 10 people in an elevator

- The mean weight of American adults is a normal random variable Y, with $\mu=170$ pounds and $\sigma=15$
- The elevator we want to install in our new office building can hold 1,800 pounds.

What is the distribution of the total weight of 10 people?

What is the probability that 10 people selected at random would exceed the elevator's weight limit?

Combinations of Random Variables

In general, for two random variables Y and Z:

Mean of Y + Z = $\mu_{Y+Z} = \mu_Y + \mu_Z$

and

Mean of Y - Z = $\mu_{Y-Z} = \mu_Y - \mu_Z$

(Note: These rules are true for discrete or continuous random variables, and are true whether or not the variables are independent)

Combinations of Random Variables

For two <u>independent</u> random variables Y and Z:

Variance of $Y + Z = \sigma^2_{Y+Z} = \sigma^2_Y + \sigma^2_Z$

and

Variance of $Y - Z = \sigma^2_{Y-Z} = \sigma^2_Y + \sigma^2_Z$

(*Note*: These rules for combinations of variance only hold for <u>independent</u> random variables, but they work for discrete or continuous variables.)

WORKSHEET

It is often useful to combine (e.g., add or subtract) random variables

Example: The weight of 10 people in an elevator

The mean weight of American adults is a normal random variable Y, with μ=170 pounds and σ=15

The elevator we want to install in our new office building can hold 1,800 pounds.

What is the distribution of the total weight of 10 people?

What is the probability that 10 people selected at random would exceed the elevator's weight limit?

WORKSHEET

It is often useful to combine (e.g., add or subtract) random variables

Example: The weight of 10 people in an elevator

The mean weight of American adults is a normal random variable Y, with μ=170 pounds and σ=15

The elevator we want to install in our new office building can hold 1,800 pounds.

What is the distribution of the total weight of 10 people?

E(Y) = 10 x 170

Variance(Y) = 10 x 15^2 = 2,250, so standard deviation = 47.4

WORKSHEET

It is often useful to combine (e.g., add or subtract) random variables

Example: The weight of 10 people in an elevator

The mean weight of American adults is a normal random variable Y, with μ=170 pounds and σ=15

The elevator we want to install in our new office building can hold 1,800 pounds.

What is the probability that 10 people selected at random would exceed the elevator's weight limit?

Z = (1800-1700)/47.4 = 2.11.

P(Z>2.11) = 0.0174