## Probability

**Random Event**

An event in which the outcome is determined by chance

**Probability**

An expression of how likely it is that a particular event (E) will result from a random event

Probabilities are expressed as a number between 0 (the event will certainly <u>not</u> occur) and 1 (the event certainly <u>will</u> occur)

P(E)=0.2 means "the probability that event X will occur is 0.2"

P(~E)=0.8 means "the probability that event X will <u>not</u> occur is 0.8"

P(E) + P(~E) = 1.0 … so … P(~E) = 1.0 – P(E)

## Probability

**Random Event**: *Tossing a Fair Coin*

*Possible Outcomes*:

*Probabilities*: P( ) = 0.5 ; P( ) = 0.5 ; P( ) + P( ) = 1.0

**Random Event**: *Roll a Pair of Dice*

*Possible Outcomes*:

*Probabilities*: P( ) = 0.167 ; P( ) = 0.167; … ; P( ) = 0.167

P( ) + P( ) + … + P( ) = 1.0

**Random Event**: *Whether a Person Has Ever Been Arrested*

*Possible Outcomes*: Has Been Arrested; Has Never Been Arrested

*Probabilities*: P(Ever Arrested) = 0.3 ; P(Never Arrested) = 0.7

P(Ever Arrested) + P(Never Arrested) = 1.0

## Probability

**Independent Random Events**

Events A and B are *independent* if P(A) is the same regardless of whether Event B happens, and P(B) is the same regardless or whether Event A happens

*Example*: If you flip two coins, P( ) = 0.5 for the second coin regardless of what happened when you flipped the first coin

**Conditional (or non-Independent) Random Events**

Event B is conditional on Event A if P(B) depends on whether or not Event A happened

*Example*: If you draw two cards from a deck of playing cards, P( ) on your second card depends on what you drew on your first card.

## Probability

**Independent Random Events**

For two independent random events A and B…

…the probability that both Event A <u>and</u> Event B happen is:

P(A and B) = P(A) x P(B)

Example: P( and ) = P( ) x P( ) = 0.167 x 0.167 = 0.028

…the probability that either Event A <u>or</u> Event B happens is:

P(A or B) = P(A) + P(B) – P(A and B)

Example: P( or ) = P( ) + P( ) – P( and )

0.167 + 0.167 – 0.028 = 0.361

## Probability

**Conditional (or non-Independent) Random Events**

If P(B) for Event B is dependent upon the outcome of Event A, then…

…the probability that both Event A <u>and</u> Event B happen is:

P(A and B) = P(A) x P(B|A)

Example: What is the probability of drawing two cards (A and B) from a deck and having them both be red? For the first drawn card, P(A) = 26/52 = 0.5. However, for the second drawn card, P(B) is not 26/52 because there are only 51 cards left in the deck. The probability that the second card is red *given that the first card was red* … P(B|A) … is 25/51 = 0.4902. So P(A and B) = P(A) x P(B|A) = 0.5 x 0.4902 = 0.2451

## Probability

If we flip a fair coin 3 times, we would observe independent events A, B, and C. What are the possible outcomes? How likely is each outcome?

P( ) = 0.5 x 0.5 x 0.5 = 0.125

P( ) = 0.5 x 0.5 x 0.5 = 0.125

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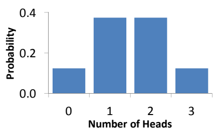
P( ) = 0.5 x 0.5 x 0.5 = 0.125

## Probability

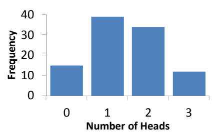
If we flip a fair coin 3 times, how many different numbers of "heads" might come up? How likely is each possible value?

P(3 Heads) = P(   ) = 0.125
P(2 Heads) = P(   ) + P(   ) + P(   ) = 3x0.125 = 0.375
P(1 Heads) = P(   ) + P(   ) + P(   ) = 3x0.125 = 0.375
P(0 Heads) = P(   ) = 0.125



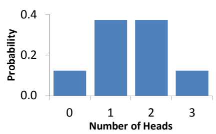

## Probability

Now imagine I went and *actually* flipped three fair coins, recorded the number of heads, and repeated that game 100 times. On the left is what I observed:




What we **saw**

This is an **observed** distribution
(from flipping 3 coins 100 times)

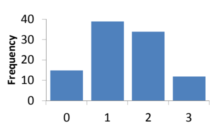



What we **expected**

This is a **theoretical** distribution
(if we flipped 3 coins ∞ times)

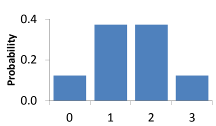
## Probability

Now imagine I went and *actually* flipped three fair coins, recorded the number of heads, and repeated that game 100 times. On the left is what I observed:




What we **saw**

This is **sample** data
(from flipping 3 coins 100 times)




What we **expected**

This is **population** data
(if we flipped 3 coins ∞ times)

## Random Variables

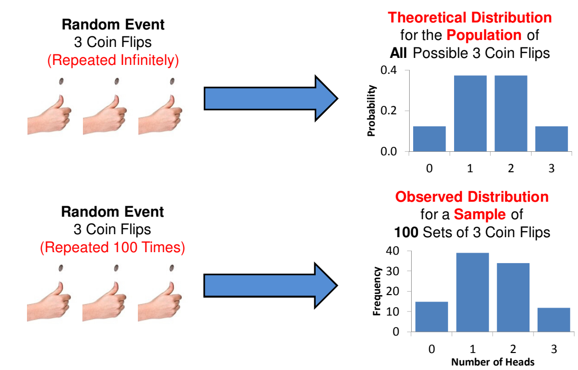
**Random variable**

The numeric outcome of a random event

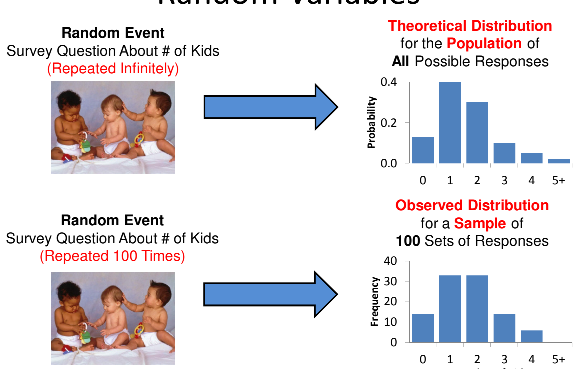
| **Random Event** | **Random Variable** |
|---|---|
| Flip a fair coin 3 times | Number of "heads" observed |
| Sample a person from the U.S. | Whether they have been arrested |
| Sample water from a lake | Bacteria per cubic inch |

For each random variable, the outcome values vary across cases as a function of a chance process, and are unknown in advance

## Random Variables




## Random Variables



## Reality    Data    Knowledge

2954462187388221348
8711102682140050851
9291758426493659357
1797402968538728539
4401671369193970834
0165785879715135279
6365488407871392124
2920353540363160907
9546090581563252173
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8159244365888845385
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4385245611536981630
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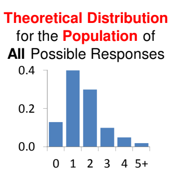
## Reality    Data    Knowledge




2954462187388221348
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0229635205640107762
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4956868092019177602
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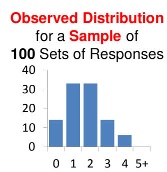



## Random Variables

**Today**

We <u>know</u> the actual probabilities associated with the random event that generates the theoretical distribution (e.g., coin flips)

We will learn to *describe* and *make practical use* of these theoretical distributions

**Later (and in Life)**

We <u>do not know</u> the actual probabilities associated with the random event (e.g., number of children per person, which candidate will win)

Our ability to *describe* and *make practical use* of theoretical distributions will allow us to <u>infer</u> those probabilities

# Random Variables

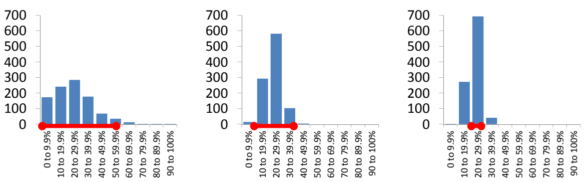
**1. Confidence Intervals**

Based on the distribution of Y in sample data, we are confident that the distribution of Y in the population has particular qualities (e.g., that its mean is within a certain range of values)

*"With 95% certainty, I conclude based on my sample data that between 25% and 35% of everyone in the population has been arrested"*

# Random Variables

**Question: What percentage of Americans were high school graduates in 1940?**




Results from asking **10** randomly selected people; trial repeated 1,000 times

$\pm \frac{1}{\sqrt{10}} \times 100\% = \pm 32\%$

Results from asking **50** randomly selected people; trial repeated 1,000 times

$\pm \frac{1}{\sqrt{50}} \times 100\% = \pm 14\%$

Results from asking **1,000** randomly selected people; trial repeated 1,000 times

$\pm \frac{1}{\sqrt{1000}} \times 100\% = \pm 3\%$

# Random Variables

**2. Hypothesis Tests**

Based on the distribution of Y in the sample data, we can evaluate the likely truth of theoretically-informed hypotheses about the distribution of Y in the population (e.g., that the mean of X is above some value)

*"With 95% certainty, I reject the claim that fewer than 20% of everyone in the population has ever been arrested"*

## Random Variables

Imagine you flip a bottle cap 100 times, and count the number of heads


If you got 52 heads out of 100, would you reject the idea that P(Heads) = 0.5?

What if you got 75 heads?

## Random Variables

**Discrete Random Variable**

Can only equal a finite number of distinct values

*Example*: When you flip a coin 3 times, you can only get four possible values … the whole numbers 0 through 3

**Continuous Random Variable**

Can take any numeric value within a range of values

*Example*: The number of miles from campus students live can take on just about any value (0.12, 1.17, 2.00, etc.)

*Note*: Discrete random variables with lots of values (e.g., number of Facebook friends) are often treated as continuous; continuous variables that are rounded (e.g., age) may seem discrete

## Discrete Random Variables

Random variable Y has k possible values

P(Y=1) + P(Y=2) + … P(Y=k) = 1.0

*Example*: If I roll a single die (the random event) and record the number of pips that come up (Y), then there are 6 possible value that might come up: ⚀ ⚁ ⚂ ⚃ ⚄ ⚅

P(Y=⚀) + P(Y=⚁) + … + P(Y=⚅) = 1.0

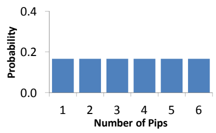
0.167 + 0.167 + 0.167 + 0.167 + 0.167 + 0.167 = 1.0

## Discrete Random Variables

*Example*: If I roll a single die (the random event) and record the number of pips that come up (Y), then there are 6 possible value that might come up:

The **probability distribution function** for Y is:

| k | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| p(Y=k) | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 |

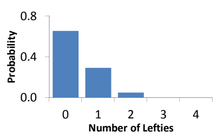



## Discrete Random Variables

*Example*: The probability that a baby is born left-handed is 0.1. For families with 4 babies, the number of left-handed babies (Y) can thus be 0, 1, 2, 3, or 4

The **probability distribution function** for Y is:

| k | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| p(Y=k) | 0.6561 | 0.2916 | 0.0486 | 0.0036 | 0.0001 |




## Discrete Random Variables

Where is the center of the theoretical distribution of discrete random variable Y?

The **expected value** of Y … called E(Y) or $\mu_Y$ … is the mean of Y in the population of all possible outcomes

One way to think about it: If you repeated the random event an infinite number of times, the expected value is the mean of Y that you would observe

## Discrete Random Variables

For discrete random variable Y:

$Y_1$, $Y_2$, $Y_3$ , … , $Y_k$ are all of the k possible values of Y

$p_1$, $p_2$, $p_3$, … , $p_k$ are the probabilities of observing each value

The **expected value** of Y … the mean of Y that you would get if you repeated the random trial an infinite number of times … is:

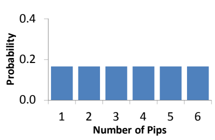
$$E(Y) = \mu_Y = \sum_{i=1}^{k} Y_i p(Y_i)$$

## Discrete Random Variables

*Example*: If I roll a single die (the random event) and record the number of pips that come up (Y), then there are 6 possible value that might come up:

The **probability distribution function** for Y is:

| k | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| p(Y=k) | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 |




$$E(Y) = \mu_Y = \sum_{i=1}^{k} Y_i p(Y_i)$$

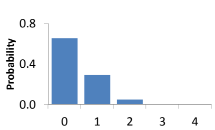
E(Y) = (1)(0.167) + (2)(0.167) + (3)(0.167) + (4)(0.167) + (5)(0.167) + (6)(0.167) = 3.5

## Discrete Random Variables

*Example*: The probability that a baby is born left-handed is 0.1. For families with 4 babies, the number of left-handed babies (Y) can thus be 0, 1, 2, 3, or 4

The **probability distribution function** for Y is:

| k | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| p(Y=k) | 0.6561 | 0.2916 | 0.0486 | 0.0036 | 0.0001 |




$$E(Y) = \mu_Y = \sum_{i=1}^{k} Y_i p(Y_i)$$

E(Y) = (0)(0.6561) + (1)(0.2916) + (2)(0.0486) + (3)(0.0036) + (4)(0.0001) = 0.4

## Discrete Random Variables

E(Y) is a measure of central tendency for the distribution of theoretical discrete variable Y

How much "spread" or variability is there?

For discrete random variable Y, the variance of Y … $\sigma^2_Y$ … is

$$\sigma_Y^2 = \sum_{i=1}^{k} (Y_i - \mu_Y)^2 p(Y_i)$$

and the standard deviation of Y … $\sigma_Y$ … is

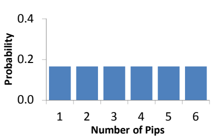
$$\sigma_Y = \sqrt{\sum_{i=1}^{k} (Y_i - \mu_Y)^2 p(Y_i)}$$

## Discrete Random Variables

*Example*: If I roll a single die (the random event) and record the number of pips that come up (Y), then there are 6 possible value that might come up:

The **probability distribution function** for Y is:

| k | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| p(Y=k) | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 | 0.167 |




$$\sigma_Y^2 = \sum_{i=1}^{k} (Y_i - \mu_Y)^2 p(Y_i)$$

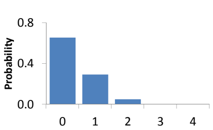
$\sigma^2_Y$ = $(1-3.5)^2(0.167) + (2-3.5)^2(0.167) + (3-3.5)^2(0.167) + (4-3.5)^2(0.167) + (5-3.5)^2(0.167) + (6-3.5)^2(0.167)$ = 2.92

## Discrete Random Variables

*Example*: The probability that a baby is born left-handed is 0.1. For families with 4 babies, the number of left-handed babies (Y) can thus be 0, 1, 2, 3, or 4

The **probability distribution function** for Y is:

| k | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| p(Y=k) | 0.6561 | 0.2916 | 0.0486 | 0.0036 | 0.0001 |




$$\sigma_Y^2 = \sum_{i=1}^{k} (Y_i - \mu_Y)^2 p(Y_i)$$

$\sigma^2_Y$ = $(0-0.4)^2(0.6561) + (1-0.4)^2(0.2916) + (2-0.4)^2(0.0486) + (3-0.4)^2(0.0036) + (4-0.4)^2(0.0001)$ = 0.36

## Discrete Random Variables

For the discrete random variable Y = "number of babies out of 4 born left handed" we found that the expected number of left handed babies was $E(Y)=\mu_Y= 0.4$, with standard deviation $\sigma^2_Y=0.36$

If we sampled an infinite number of families with four kids and observed Y for each, the mean of Y would be 0.4

We wouldn't always (or ever) see exactly 0.4 left-handed kids; $\sigma^2_Y$ indicates how much the number of left-handed kids would vary from family to family

## Worksheet

Below is a probability distribution function for discrete variable Y. Compute the mean and standard deviation of this distribution

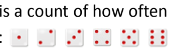
| *k* | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| *P*(*Y*=*k*) | 0.8 | 0.1 | 0.05 | 0.05 |

## Binomial Random Variables

A special kind of discrete random variable … called a binomial random variable … is one in which the random event generating it has only two possible outcomes

**Discrete Random Variable**

Random event has a finite number of distinct values; the random variable is a count of how often each value occurs.

*Example*: 

**Binomial Random Variable**

Random event has exactly two possible values ("success" and "failure"); the random variable is a count of how often "success" occurs over *n* repeated trials. *Example*: 

## Binomial Random Variables

Binomial random variable Y equals the number of "successes" out of n binomial events when the probability of "success" in any one event is p

The probability that Y = k (where k ranges from 0 to n) is:

$$P(Y=k)=\frac{n!}{k!(n-k)!}p^{k}(1-p)^{n-k}$$

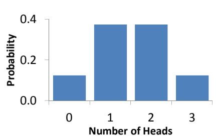
For example, what is the probability of getting k=2 heads out of n=3 flips of a fair coin where P(Heads)=0.5?

$$P(Y=2)=\frac{3!}{2!(3-2)!}0.5^{2}(1-0.5)^{3-2}=\frac{6}{2\,x\,1}(0.25)(0.5)$$
$$=0.375$$

## Binomial Random Variables

Applying this formula to each possible value of Y when n=3 and p=0.5, we get the probability distribution function:

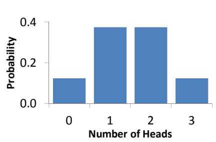
| *k* | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| *P*(*Y*=*k*) | 0.125 | 0.375 | 0.375 | 0.125 |




## Binomial Random Variables

But we already knew this from before…

P(3 Heads) = P( ) = 0.125
P(2 Heads) = P( ) + P( ) + P( ) = 3x0.125 = 0.375
P(1 Heads) = P( ) + P( ) + P( ) = 3x0.125 = 0.375
P(0 Heads) = P( ) = 0.125



## Binomial Random Variables

For binomial random variable Y based on n trials with probability of "success" on any given trial equal to p, the expected value of Y (or $\mu_Y$) equals

$$E(Y) = \mu_Y = np$$

the variance $\sigma^2_Y$ equals

$$\sigma^2_Y = np(1-p)$$

and the standard deviation $\sigma_Y$ equals

$$\sigma_Y = \sqrt{\sigma^2_Y} = \sqrt{np(1-p)}$$

## Binomial Random Variables

Because a binomial random variable is just a special case of discrete random variables, the formula used to compute the mean and variance of a discrete random variable…

$$E(Y) = \mu_Y = \sum_{i=1}^{k} Y_i p(Y_i)$$

$$\sigma^2_Y = \sum_{i=1}^{k} (Y_i - \mu_Y)^2 p(Y_i)$$

…will also work for binomial random variables (if you are inclined to do extra math)

## Binomial Random Variables

Whether a newborn baby is a boy is a binomial experiment with p = 0.512

For the binomial random variable "number of boys" based on n = 100 randomly selected births:

$$E(Y) = \mu_Y = np = 100 \text{ x } 0.512 = 51.2$$

$$\sigma^2_Y = np(1-p) = 100(0.512)(1 - 0.512) = 24.99$$

$$\sigma_Y = \sqrt{\sigma^2_Y} = \sqrt{np(1-p)} = \sqrt{24.99} = 4.999$$

If we sampled 100 births and counted the number of boys … and then repeated that experiment an infinite number of times…we would average 51.2 boys with $\sigma_Y$ = 4.999

## Worksheet

The probability that a new car is a "lemon" is 0.01

A factory produces 8 cars per hour

- What is the probability that the factory will produce exactly zero "lemons" in a given hour?
- What is the mean and standard deviation of the number of "lemons" the factory produces per hour?

## Want More?

This is a good reading:

- http://www.stat.auckland.ac.nz/~wild/ChanceEnc/Ch05.pdf

David Lane's Book

- http://onlinestatbook.com/2/probability/probability.html

Gerard Dallal's Book

- http://www.jerrydallal.com/LHSP/prob.htm

**BREAK**

## Random Variables

**Discrete Random Variable**

Can only equal a finite number of distinct values

*Example*: When you flip a coin 3 times, you can only get four possible values … the whole numbers 0 through 3

**Continuous Random Variable**

Can take any numeric value within a range of values

*Example*: The number of miles from campus students live can take on just about any value (0.12, 1.17, 2.00, etc.)

*Note*: Discrete random variables with lots of values (e.g., number of Facebook friends) are often treated as continuous; continuous variables that are rounded (e.g., age) may seem discrete

## Random Variables

**Today**

We know the actual probabilities associated with the random event that generates the theoretical distribution (e.g., coin flips)

We will learn to *describe* and *make practical use* of these theoretical distributions

**Later (and in Life)**

We do not know the actual probabilities associated with the random event (e.g., number of children per person, which candidate will win)

Our ability to *describe* and *make practical use* of theoretical distributions will allow us to infer those probabilities

## Continuous Random Variables

For discrete random variables we began by computing the probability of observing each possible outcome

We can't do this for continuous random variables because there are (by definition) an infinite number of possible outcomes

Instead of determining the probability that *Y* equals particular values and specifying a probability distribution function, we determine the probability that *Y* falls within a certain range of the probability density function

## Continuous Random Variables

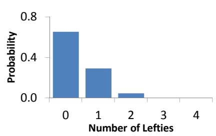
For the discrete random variable *Y*, the <u>probability distribution function</u> reports the probability of observing each possible value of *Y*

For a continuous random variable, the <u>probability density function</u> is a curve that provides information about the probability that *Y* falls between two values *a* and *b*

P(a ≤ Y ≤ b) is the area under the curve over the interval between the values a and b

## Continuous Random Variables

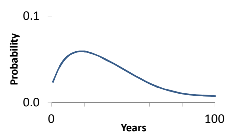
**Probability Distribution Function**
Number Left Handed
Out of Four Babies




*Example*: P(Y=1) = 0.2916

*"The probability that Y equals 1 is 0.2916"*

**Probability Density Function**
Years from Birth
until Death

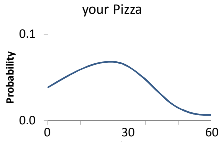



*Example*: P(20≤Y≤40) = 0.40

*"The probability that you live between 20 and 40 years is 0.40"*

## Continuous Random Variables

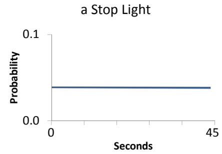
**Probability Density Function**
Time Waiting for
your Pizza




*Example*: P(30≤Y) = 0.70

*"The probability that you wait 30 minutes or longer is 0.70"*

**Probability Density Function**
Time Waiting at
a Stop Light




*Example*: P(0≤Y≤45) = 1.00

*"The probability that you wait between 0 and 45 seconds is 1.00"*

## Continuous Random Variables

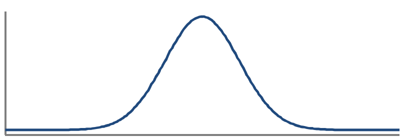
As before, these are theoretical distributions … distributions of Y if we were to sample from the population an infinite number of times

As with all distributions, we can describe distributions with respect to their central tendency and amount of variability

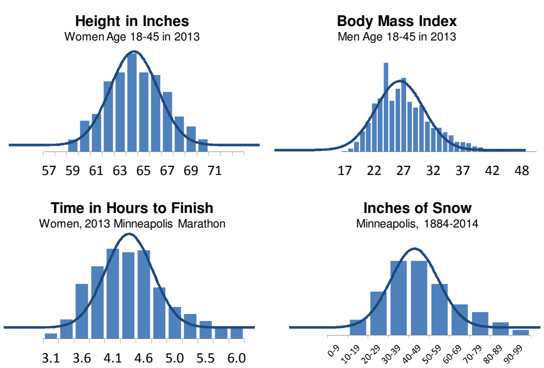


## Normal Random Variables

Just as a binomial random variables is a special (and very common) kind of discrete random variable, a **normal random variables** is a special (and very common) kind of continuous random variable



## Normal Random Variables



## Normal Random Variables

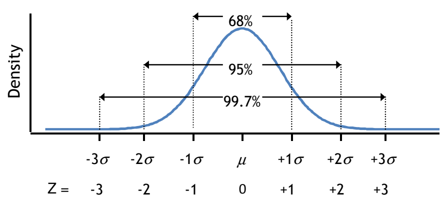
Any normal random variable Y is symmetric and can be characterized by its mean $\mu_Y$ and standard deviation $\sigma_Y$

Remember Z scores? $Z = \frac{(Y - \bar{Y})}{s_Y}$

For *any* normally distributed random variable:

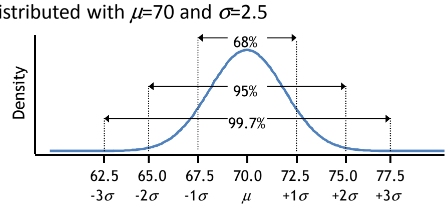
- ~68% of cases fall within the range -1Z and +1Z
- ~95% of cases fall within the range -2Z and +2Z
- ~99.7% of cases fall within the range -3Z and +3Z
- 100% of cases fall within the range $-\infty$Z and $+\infty$Z

## Normal Random Variables




## Normal Random Variables

Men's height in inches in the United States is normally distributed with $\mu$=70 and $\sigma$=2.5



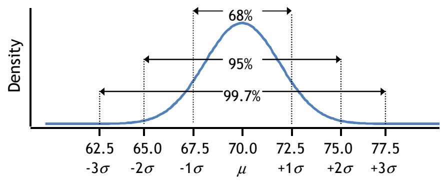

**What proportion of men is more than 75" tall?**

$Z = (Y - \mu_Y)/\sigma_Y = (75-70)/2.5 = 2$

$P(Z \le 2) = 0.975$ and so $P(Z>2) = 1 - 0.975 = 0.025$

## Normal Random Variables

Men's height in inches in the United States is normally distributed with $\mu$=70 and $\sigma$=2.5



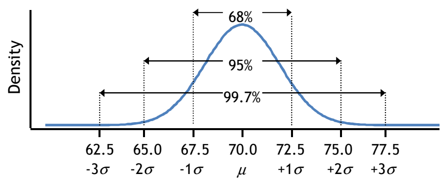

**What proportion of men is between 67.5" and 75" tall?**

$Z_{75}$ = (75–70)/2.5 = 2 ; $Z_{67.5}$ = (67.5–70)/2.5 = -1

P(67.5"≤ Z ≤ 75") = P(Z≤2) — P(Z≤-1) = 0.975 — 0.160 = 0.815

## Worksheet

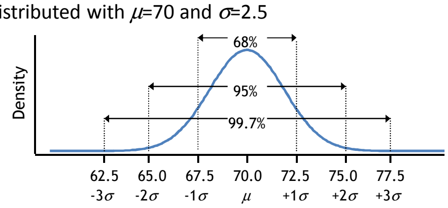
Men's height in inches in the United States is normally distributed with $\mu$=70 and $\sigma$=2.5




**What proportion of men is taller than me (69.5")?**

## Worksheet

Men's height in inches in the United States is normally distributed with $\mu$=70 and $\sigma$=2.5




What proportion of men is between 66" and 71" tall?

## Worksheet

The distributions of times for 19-34 year men and women to complete a half marathon are (more or less) normal

**Men**: $\mu_{Y_{MEN}}$ (in minutes) is 125 with $\sigma_{Y_{MEN}}$ of 25
**Women**: $\mu_{Y_{WOMEN}}$ is 140 with $\sigma_{Y_{WOMEN}}$ of 23

What proportion of women finish half marathons ahead of the average man?

(Change of Topic)

## Combinations of Random Variables

It is often useful to combine (e.g., add or subtract) random variables

*Example*: Time to commute to and from work

- The time that it takes to drive to work is a normal random variable $Y_{TO}$, with $\mu_{TO}$=30 minutes and $\sigma_{TO}$=10
- The time that it takes to drive home from work is also a normal random variable $Y_{FROM}$, with $\mu_{FROM}$=25 and $\sigma_{FROM}$=15

What is the distribution of total commute time?

What is the distribution of the difference between commutes to and from work?

## Combinations of Random Variables

In general, for two random variables Y and Z:

Mean of Y + Z = $\mu_{Y+Z} = \mu_Y + \mu_Z$

and

Mean of Y - Z = $\mu_{Y-Z} = \mu_Y - \mu_Z$

(Note: These rules are true for discrete or continuous random variables, and are true whether or not the variables are independent)

## Combinations of Random Variables

For two independent random variables *Y* and *Z*:

Variance of *Y* + *Z* = $\sigma^2_{Y+Z} = \sigma^2_Y + \sigma^2_Z$

and

Variance of *Y* - *Z* = $\sigma^2_{Y-Z} = \sigma^2_Y + \sigma^2_Z$

(*Note*: These rules for combinations of variance only hold for independent random variables, but they work for discrete or continuous variables.)

## Combinations of Random Variables

*Example*: Time to commute to and from work

- The time that it takes to drive to work is a normal random variable $Y_{TO}$, with $\mu_{TO}$=30 minutes and $\sigma_{TO}$=10
- The time that it takes to drive home from work is also a normal random variable $Y_{FROM}$, with $\mu_{FROM}$=25 and $\sigma_{FROM}$=15

What is the distribution of total commute time?

$\mu_{TO+FROM} = \mu_{TO} + \mu_{FROM} = 30 + 25 = 55$ minutes

$\sigma^2_{TO+FROM} = \sigma^2_{TO} + \sigma^2_{FROM} = 10^2 + 15^2 = 325$

## Combinations of Random Variables

*Example*: Time to commute to and from work

The time that it takes to drive to work is a normal random variable $Y_{TO}$, with $\mu_{TO}$=30 minutes and $\sigma_{TO}$=10

The time that it takes to drive home from work is also a normal random variable $Y_{FROM}$, with $\mu_{FROM}$=25 and $\sigma_{FROM}$=15

What is the distribution of the difference between commutes to and from work?

$$\mu_{TO\text{-}FROM} = \mu_{TO} - \mu_{FROM} = 30 - 25 = 5 \text{ minutes}$$
$$\sigma^2_{TO\text{-}FROM} = \sigma^2_{TO} + \sigma^2_{FROM} = 10^2 + 15^2 = 325$$

## Worksheet

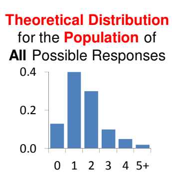
The distributions of times for 19-34 year men and women to complete a half marathon are (more or less) normal

**Men**: $\mu_{Y_{MEN}}$ (in minutes) is 125 with $\sigma_{Y_{MEN}}$ of 25

**Women**: $\mu_{Y_{WOMEN}}$ is 140 with $\sigma_{Y_{WOMEN}}$ of 23

What is the distribution of the difference between men's and women's race times?

## Reality Data Knowledge

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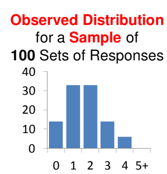

## Random Variables

**Today**

We <u>know</u> the actual probabilities associated with the random event that generates the theoretical distribution (e.g., coin flips)

We will learn to *describe* and *make practical use* of these theoretical distributions

**Later (and in Life)**

We <u>do not know</u> the actual probabilities associated with the random event (e.g., number of children per person, which candidate will win)

Our ability to *describe* and *make practical use* of theoretical distributions will allow us to <u>infer</u> those probabilities

## Want More?

This is a good reading:

http://www.stat.cmu.edu/~cshalizi/36-220/lecture-7.pdf

David Lane's Book

http://onlinestatbook.com/2/normal_distribution/normal_distribution.html

Gerard Dallal's Book

http://www.jerrydallal.com/LHSP/normal.htm