*YOUR TA’S NAME*:

*Lecture Worksheet*

*Tuesday 9/29/2020*

**MAIN POINTS OF LECTURE**

1. Definitions:
	1. A random event is an event in which the outcome is determined by chance (e.g., whether a flipped coin will turn up heads or tails)
	2. Probability is an expression of how likely it is that a particular outcome will result from a random event where 0 means it certainly won’t happen and 1 means it certainly will (e.g., 0.5 for heads and 0.5 for tails if you flip a fair coin)
	3. A random variable is the numeric outcome of a random event (e.g., how many heads turn up if you flip a fair coin 10 times)
2. A discrete random variable (*Y*) has a finite (*k*) number of possible values. P(Y=j) is the probability that Y equals the jth value (out of k)
	1. A probability distribution function is a table that lists the *k* possible values of *Y* and the probability that each of those *k* values is observed. The sum of P(Y=j) across the *k* values must be 1.0
	2. The expected value of discrete variable *Y* … or the center of the probability distribution function … or the average value we would expect if we repeatedly observed values of *Y* … is  (*Equation 7 in your syllabus*)
	3. The standard deviation of that discrete random variable is

 (*Equation 8 in your syllabus*)

1. A binomial random variable is a type of discrete random variable which is a count of how many times an event occurs within a given number of trials (e.g., how many heads turn up out of 10 flips of a fair coin).
	1. Let *n* be the number of trials, and for each trial define “success” or “failure” in whatever way makes sense. P(success) must be the same across all *n* trials
	2. If there are *k* possible values of binomial random variable *Y*, then the probability of observing any one of them is (*Equation 9 in your syllabus*)
	3. The expected value of binomial random variable *Y* … or the center of the probability distribution function … or the average value we would expect if we repeatedly observed values of *Y* … is  (*Equation 10 in your syllabus*)
	4. The standard deviation of that binomial random variable is

 (*Equation 11 in your syllabus*)

* 1. Because binomial random variables are a special case of discrete random variables, the formulas for the expected value and standard deviation of *all* discrete random variables (Equations 7 and 8 in your syllabus) will also work for binomial random variables. They just require lots more work.

**QUESTIONS**

1. (From the recorded lecture): Whether a job applicant gets the job is a random event. If you pick two people from among the applicants for a particular job, and only one of them can get the job, are their application outcomes independent or dependent events?

They are not independent events. If one person gets the job, the other person’s odds of getting the job go down (to zero). If one person does not get the job, the other person’s odds are higher.

1. (From the recorded lecture): Below is the probability distribution function for discrete variable Y

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| *k* | 0 | 1 | 2 | 3 |
| *P*(*Y*=*k*) | 0.8 | 0.1 | 0.05 | 0.05 |

Compute the mean and standard deviation of this distribution

MEAN: 

 = (0)(0.8) + (1)(0.1) + (2)(0.05) + (3)(0.05) = 0.35

STANDARD DEVIATION: 

= Square root of [ (0-0.35)2(0.8) + (1-0.35)2(0.1) + (2-0.35)2(0.05) + (3-0.35)2(0.05) ]

= Square root of [ (0.098) + (0.04225) + (0.136125) + (0.351125) ]

= Square root of 0.6275

= 0.792

1. (From the recorded lecture): The probability that a new car is a “lemon” is 0.01. A factory produces 8 cars per hour.

What is the probability that the factory will produce exactly zero “lemons” in a given hour?

$$P(Y=0) = \frac{n!}{k!\left(n-k\right)!}p^{k}(1-p)^{n-k}= \frac{8!}{0!\left(8-0\right)!}0.01^{0}\left(1-0.01\right)^{8-0}=\left(1\right)\left(1\right)\left(0.923\right)=0.923$$

What is the mean and standard deviation of the number of “lemons” the factory produces per hour?

E(Y) = np = 8 x 0.01 = 0.08

Standard deviation = square root of np(1-p) = square root of 8 x 0.01 x 0.99 = 0.281

1. (From the synchronous class session): The probability that your personal income taxes get audited by the IRS is 0.04 in any given tax year.
	1. Across 5 tax years, what is the probability that you get audited exactly twice?

$$P(Y=0) = \frac{n!}{k!\left(n-k\right)!}p^{k}(1-p)^{n-k}= \frac{5!}{2!\left(5-2\right)!}0.04^{2}\left(1-0.04\right)^{5-2}=\left(10\right)\left(0.0016\right)\left(0.885\right)=0.014$$

* 1. What is the expected value of the number of times you will be audited over those 5 years?

E(Y) = np = 5 x 0.04 = 0.2

* 1. What is the standard deviation of the number of times you will be audited over those five years?

Standard deviation = square root of np(1-p) = square root of 5 x 0.04 x 0.96 = 0.438