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**Problem Set #2**

1. The distribution of the variable Y has a mean of 10 and a standard deviation of 4.

Compute the value of Y for cases that have the following Z scores

Z = 2 Y = (Y - 10)/4 = 2, so Y - 10 = 8, so Y = 18

Z = -1.5 Y = (Y - 10)/4 = -1.5, so Y - 10 = -6, so Y = 4

Z = 1.23 Y = (Y - 10)/4 = 1.23, so Y - 10 = 4.92, so Y = 14.92

Compute the Z score for cases that have the following values of Y

Y = 12 Z = (12-10)/4 = 0.5

Y = 7 Z = (7-10)/4 = -0.75

Y = 21 Z = (21-10)/4 = 2.75

1. True story: I was at the Minnesota State Fair’s “Miracle of Birth Center” watching a sow give birth to piglets. She had just given birth to her 18th piglet. The farmer noted over the loudspeaker that sows typically give birth to only 10 piglets, with a standard deviation of 3 piglets. Given these attributes of sows, compute the Z-score corresponding to giving birth to 18 piglets.

ANSWER: Z = (X – X-bar)/sX = (18 – 10) / 3 = 2.667.

1. If you learned that your score on an exam was 80 and the mean was 70, would you be more satisfied if the standard deviation was 5 or if it was 15? Explain.

ANSWER: If I scored 10 points above the mean and the standard deviation of exam scores equals 5, then I scored 2 standard deviation above the mean—because Z would equal (80-70)/5=2. That's pretty exceptional. However, if I scored 10 points above the mean and the standard deviation equals 15, then I only scored 2/3 of a standard deviation above the mean. Not so exceptional. Then, my score would be above average but not terrible high compared to many other students. So, I’d rather have the standard deviation equal 5.

*A student is doing a senior project on students’ opinions about the quality of food served in university dining halls. So she does the following: First, she gets a list of all 12,283 students who are currently signed up for university meal plans; the list includes students’ names and email addresses. Second, she uses a random number table to randomly select 200 students from that list. Third, she sends an email to those 200 students. The email includes a link to a web survey, and in the email she explains the purpose of the survey and encourages them to participate. The web survey contains a number of questions about the food in university dining halls. Fourth, she waits for responses; after a week, she sends a reminder email to students who had not yet completed the survey. In the end, 112 students complete the survey. Fifth, she analyzes the data and writes her senior project.*

1. What is the population being studied?

ANSWER: Students who are currently signed up for university meal plans. There are 12,2832 of them.

1. What is the sample that the student selected?

ANSWER: She selected 200 of those students.

1. The student concludes that 45% of the 112 students she surveyed are satisfied with the food served in the university dining halls. What is a conservative margin of error for that estimate? How do you interpret that margin of error?

ANSWER: The conservative margin of error is 0.45±1/sqrt(112) = 0.450±0.094. Thus the confidence interval is 0.356 to 0.544. That is, because of sampling error, the student’s estimate (45%) is off by plus or minus 9.4%. Thus she can say with 95% certainty that the true population percentage of students who are satisfied with the food served in the university dining halls is between 35.6% and 54.4%. That’s a pretty wide margin of error. And, that inference assumes that there is no *systematic* sampling error … which there almost certainly is.

*The discrete random variable “Number of Times Adult Americans Have Ever Been Married” can take on the values 0 through 4 (where “4” really means “4 or more” … but ignore that for now). Here is the probability distribution function for this discrete random variable, Y:*

*k* 0 1 2 3 4

*P*(*Y*=*k*) 0.20 0.65 0.10 0.04 0.01

1. If you pick a person at random from this population, what is the probability that they will have been married more than once?

ANSWER: P(Y>1) = P(Y=2)+P(Y=3)+P(Y=4) = 0.10+0.04+0.01 = 0.15

1. What is the expected value of Y, and how do you interpret that expected value?

ANSWER: , so

E(Y) = (0)(0.20) + (1)(0.65) + (2)(0.10) + (3)(0.04) + (4)(0.01)

E(Y) = 0 + 0.65 + 0.20 + 0.12 + 0.04 = 1.01

This means that we expect, on average, to find that people have been married 1.01 times

1. What is the standard deviation of the discrete random variable Y?

ANSWER: , so

0.742

*The probability that a child is born left-handed is 0.10, and does not depend on whether his or her older siblings were left-handed. The following questions pertain to these data. For each question, be sure to indicate how you arrived at your answer.*

1. Among families that have three children, construct the probability distribution function for the discrete random variable *X*: The number of children out of three that are left-handed.

ANSWER: For each possible value of *Y*, *P*(*Y*=*k*) equals , where *n*=3, *k* equals one of the four possible outcomes (0, 1, 2, or 3), and *p*=0.10 (as given in the question). Doing this for each value of *k*, the probability distribution function is

*k* 0 1 2 3

*P*(*Y*=*k*) 0.729 0.243 0.027 0.001

1. Among families that have three children, what is the expected value of *Y*?

ANSWER: The expected value of a binomial random variable equals *np*, or (3)(0.1)=0.3.

Of course, binomial random variables are just one special kind of discrete random variable, so the (more complicated and computationally challenging) formula for the expected value of any discrete random variable will work here, too: The expected value for a discrete random variable equals , so in this case:

=0.3 … the same answer.

1. Among families that have three children, what is the standard deviation of *Y*?

ANSWER: For a binomial random variable, the standard deviation equals = =0.52

Again, binomial random variables are just one special kind of discrete random variable, so the (more complicated and computationally challenging) formula for the standard deviation of any discrete random variable will work here, too: The standard deviation for a discrete random variable equals , so in this case =

 =0.52. Same answer.